Estimations procedures with RATS 8.0 or higher for Introduction to time series and forecasting, book by Brockwell & Davis (2 ^{ed})

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Abstract

This file consists of estimations in the time series software RATS (Regression Analysis of Time Series). In this file program files and procedures for the estimations in the book by Brockwell and Davis (2 ^{ed}) had been provided. The order of estimations is by the book chapters.

Keywords: time series, forecasting, RATS 8.0, Box-Jenkins, state space models

```
open data uspop.dat

calendar(ypp=10) 1790

data(format=free,org=columns) 1790:1 1990:1 uspop


print

  ENTRY          USPOP

  1790:01        3929214

  1800:01        5308483

  1810:01        7239881

  1820:01        9638453

  1830:01       12860702

  1840:01       17063353

  1850:01       23191876

  1860:01       31443321

  1870:01       38558371

  1880:01       50189209

  1890:01       62979766

  1900:01       76212168

  1910:01       92228496

  1920:01      106021537

  1930:01      123202624

  1940:01      132164569

  1950:01      151325798

  1960:01      179323175

  1970:01      203302031

  1980:01      226542203

  1990:01      248709873

* Rescale to millions to keep the numbers more manageable
```

```
set uspop = uspop*1.e-6

* These create the trend and squared trend. T is a reserved variable

* used in SET instructions and elsewhere to represent the current time

* period.


    print

  ENTRY         USPOP

  1790:01        3.929214

  1800:01        5.308483

  1810:01        7.239881

  1820:01        9.638453

  1830:01       12.860702

  1840:01       17.063353

  1850:01       23.191876

  1860:01       31.443321

  1870:01       38.558371

  1880:01       50.189209

  1890:01       62.979766

  1900:01       76.212168

  1910:01       92.228496

  1920:01      106.021537

  1930:01      123.202624

  1940:01      132.164569

  1950:01      151.325798

  1960:01      179.323175

  1970:01      203.302031

  1980:01      226.542203

  1990:01      248.709873
```

LINREG does a linear regression. The parameter on the first line is the

```
* dependent variable. The explanatory variables are shown on a separate
* line that starts with a "#" symbol.
linreg uspop
# constant trend trend2
Linear Regression - Estimation by Least Squares
Dependent Variable USPOP
10-Year Data From 1790:01 To 1990:01
Usable Observations                     21
Degrees of Freedom                      18
Centered R^2                     0.9988833
R-Bar^2                          0.9987592
Uncentered R^2                   0.9995042
Mean of Dependent Variable    85.782623952
Std Error of Dependent Variable  78.542874302
Standard Error of Estimate     2.766660056
Sum of Squared Residuals      137.77934160
Regression F(2,18)               8050.3864
Significance Level of F          0.0000000
Log Likelihood                    -49.5496
Durbin-Watson Statistic             1.1924

    Variable                     Coeff      Std Error      T-Stat      Signif
*************************************************************************************
1.  Constant                   6.957920304  1.998525596      3.48153  0.00266376
2.  TREND                     -2.159869892  0.418437368     -5.16175  0.00006551
3.  TREND2                     0.650633863  0.018472061     35.22259  0.00000000

* PRJ (in its simplest use) gets the fitted values from the regression
* just completed.
```

```
prj fitted
graph(footer="Figure 1.8 Population of U.S. with Quadratic Trend",vlabel="Millions",$
  overlay=dots,ovsame) 2
# fitted
# uspop
```

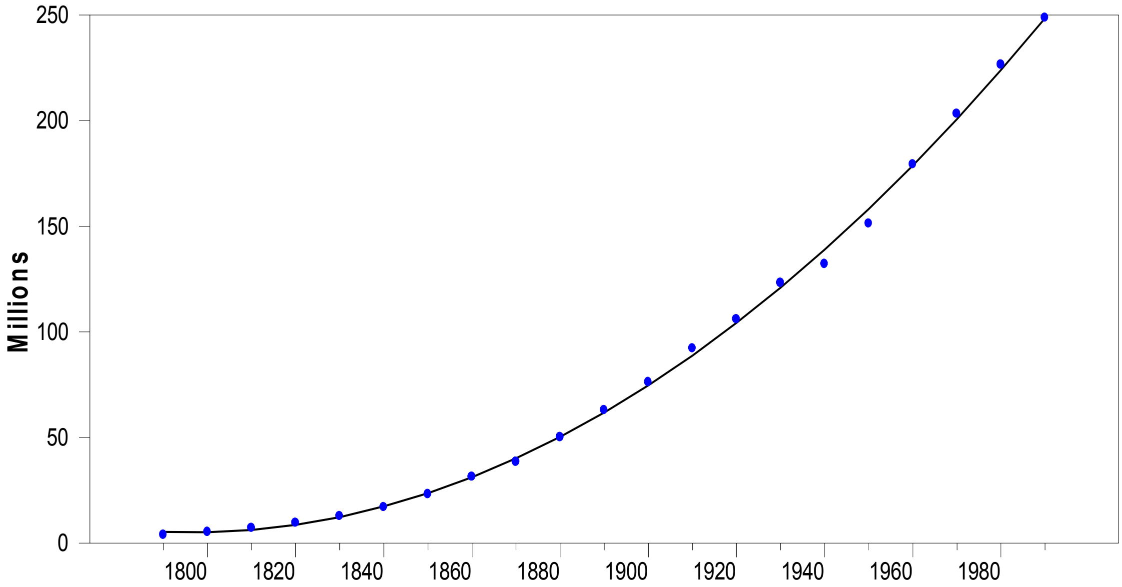


Figure 1.8 Population of U.S. with Quadratic Trend

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 1.3.5 from pp 10-12
open data lake.dat
calendar 1875
data(format=free,org=columns) 1875:1 1972:1 lake
set trend = t
linreg lake
# constant trend
prj fitted
```

```
Linear Regression - Estimation by Least Squares
```

```
Dependent Variable LAKE
Annual Data From 1875:01 To 1972:01
Usable Observations                        98
Degrees of Freedom                         96
Centered R^2                        0.2724728
R-Bar^2                             0.2648943
Uncentered R^2                      0.9848844
Mean of Dependent Variable       9.0040816327
Std Error of Dependent Variable  1.3182985260
Standard Error of Estimate       1.1302867788
Sum of Squared Residuals         122.64462743
Regression F(1,96)                    35.9538
Significance Level of F             0.0000000
Log Likelihood                      -150.0478
Durbin-Watson Statistic                0.4395

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  Constant                      10.20203661   0.23011125     44.33524  0.00000000
2.  TREND                         -0.02420111   0.00403611     -5.99615  0.00000004

graph(footer="Figure 1-9 Level of Lake Huron Showing Least Squares Fit") 2
# lake
# fitted
```

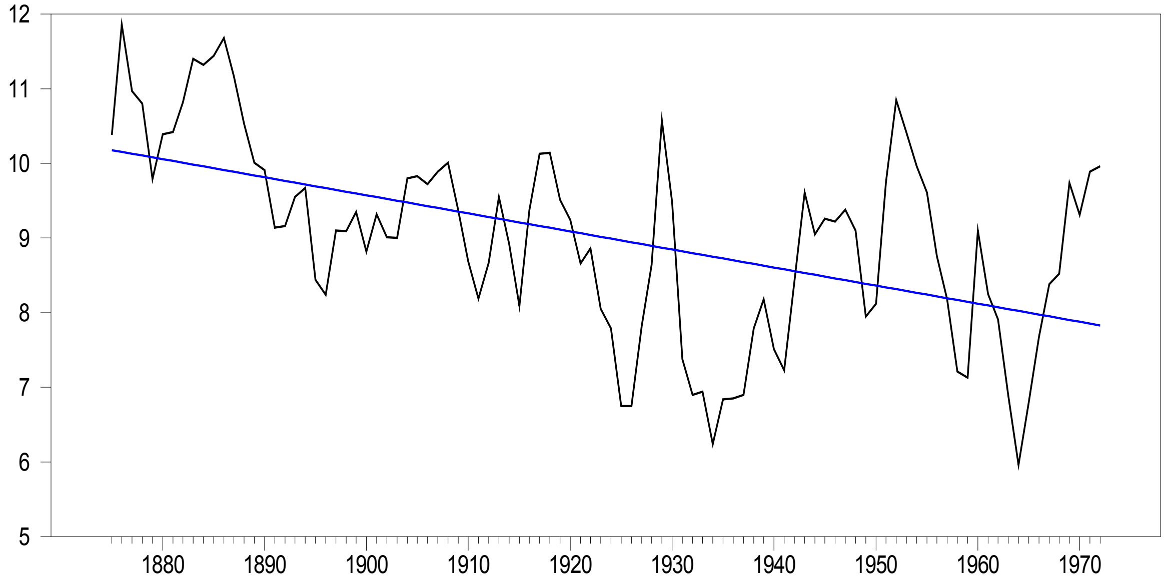


Figure 1-9 Level of Lake Huron Showing Least Squares Fit

```
* The series %RESIDS has the residuals from the most recent regression
set y = %resids
graph(footer="Figure 1-10 Residuals from Fitting a Line to Lake Data")
# y
```

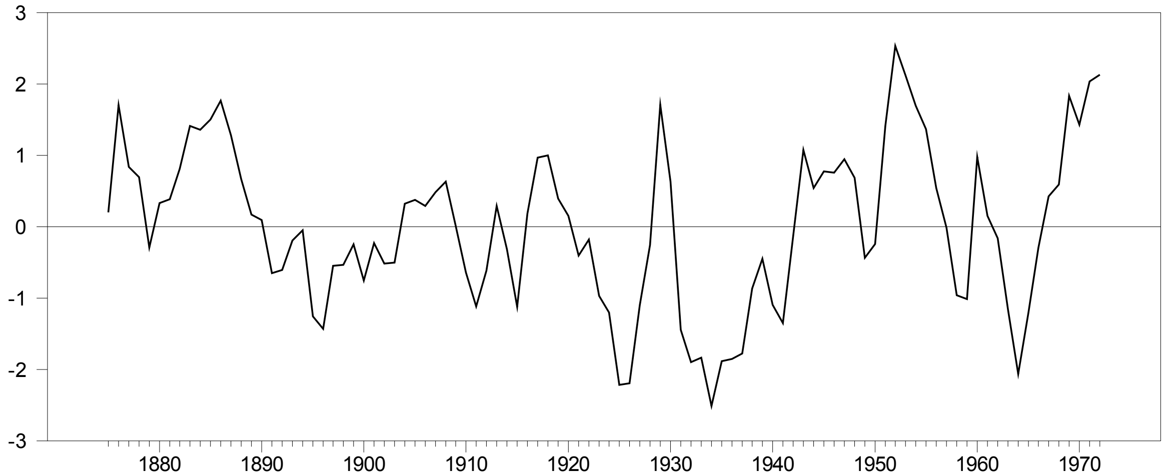


Figure 1-10 Residuals from Fitting a Line to Lake Data

```
Example 1.3.6 from pp 13-14

open data deaths.dat

calendar(m) 1973

data(format=free,org=columns) 1973:1 1978:12 deaths

source bdtrends.src
```

ENTRY	DEATHS
1973:01	9007
1973:02	8106
1973:03	8928
1973:04	9137
1973:05	10017
1973:06	10826
1973:07	11317
1973:08	10744
1973:09	9713
1973:10	9938
1973:11	9161
1973:12	8927

1974:01	7750
1974:02	6981
1974:03	8038
1974:04	8422
1974:05	8714
1974:06	9512
1974:07	10120
1974:08	9823
1974:09	8743
1974:10	9129
1974:11	8710
1974:12	8680
1975:01	8162
1975:02	7306
1975:03	8124
1975:04	7870
1975:05	9387
1975:06	9556
1975:07	10093
1975:08	9620
1975:09	8285
1975:10	8433
1975:11	8160
1975:12	8034
1976:01	7717
1976:02	7461
1976:03	7776
1976:04	7925
1976:05	8634

1976:06	8945
1976:07	10078
1976:08	9179
1976:09	8037
1976:10	8488
1976:11	7874
1976:12	8647
1977:01	7792
1977:02	6957
1977:03	7726
1977:04	8106
1977:05	8890
1977:06	9299
1977:07	10625
1977:08	9302
1977:09	8314
1977:10	8850
1977:11	8265
1977:12	8796
1978:01	7836
1978:02	6892
1978:03	7791
1978:04	8129
1978:05	9115
1978:06	9434
1978:07	10484
1978:08	9827
1978:09	9110
1978:10	9070

```
 1978:11         8633

 1978:12         9240
```

The procedure HarmonicFit performs the harmonic regression using the

```
* listed set of periodicities, producing as output the fitted values.

@harmonicfit(periods=||12,6||) deaths / hdeaths
```

* The graph in the text uses two different styles to show the actual data (squares) and fitted components (lines). The default behavior of GRAPH is to use distinct line patterns or colors instead. To mimic the appearance in the text, you can use the pair of option OVERLAY=DOTS and OVSAME. Overlay graphs do a pair of graphs in the same box. OVSAME requests that they use the same vertical scale, which is what we want here.

```
graph(footer="Figure 1-11 Estimated Harmonic Component of Accidental Deaths",$

   overlay=dots,ovsame) 2

# hdeaths

# deaths
```

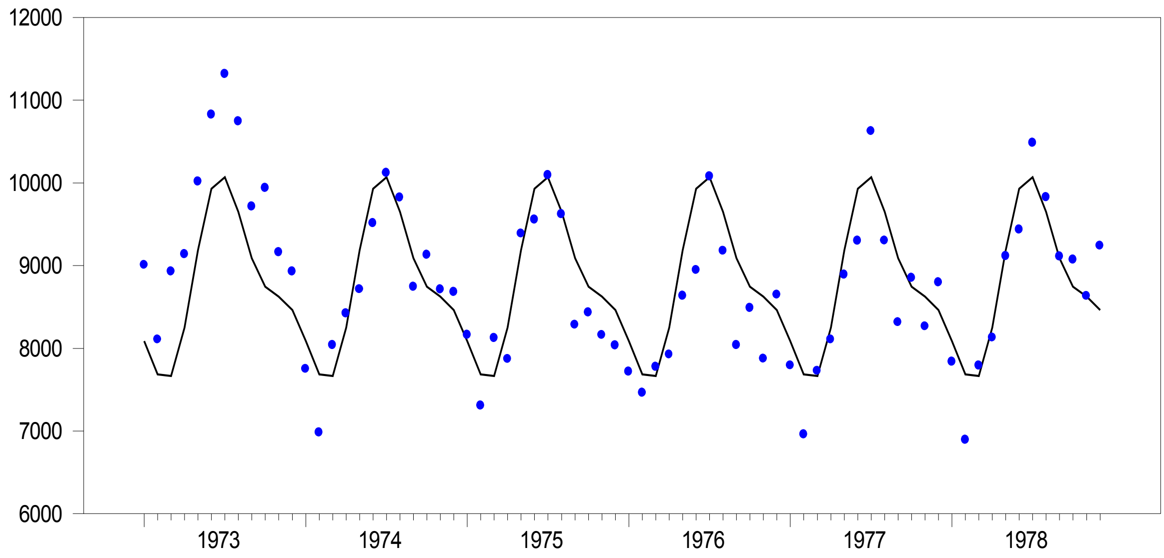


Figure 1-11 Estimated Harmonic Component of Accidental Deaths

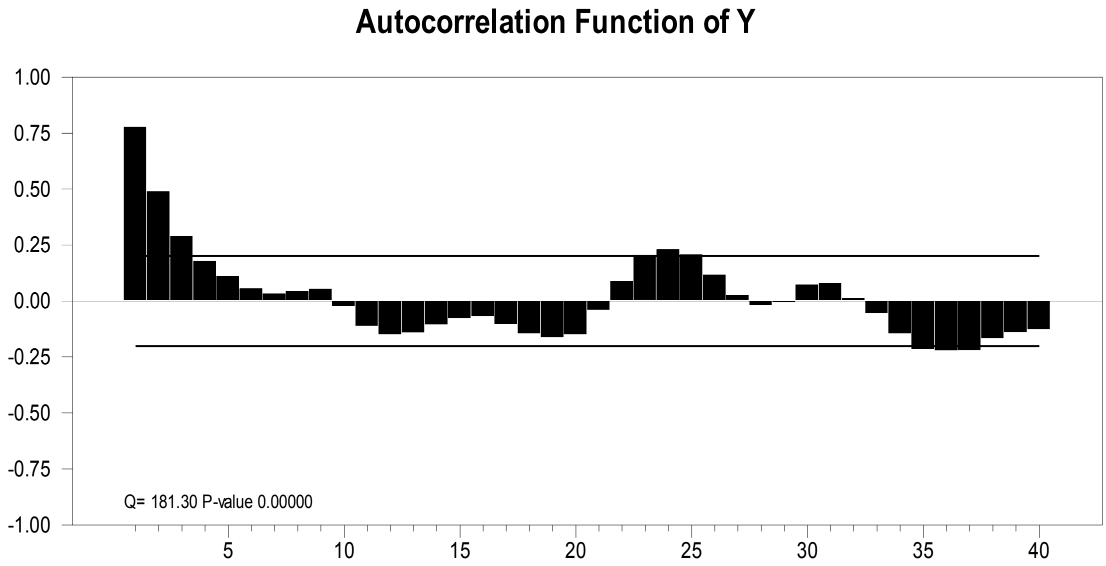
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

```
* Example 1.4.6 from page 20

set x 1 200 = %ran(1.0)

* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 1.4.2 from pp 21-23
```

```
*
open data lake.dat
calendar 1875
data(format=free,org=columns) 1875:1 1972:1 lake

set trend = t
linreg lake
# constant trend
```

```
Linear Regression - Estimation by Least Squares
Dependent Variable LAKE
Annual Data From 1875:01 To 1972:01
Usable Observations                     98
Degrees of Freedom                      96
Centered R^2                     0.2724728
R-Bar^2                          0.2648943
Uncentered R^2                   0.9848844
Mean of Dependent Variable    9.0040816327
Std Error of Dependent Variable  1.3182985260
Standard Error of Estimate    1.1302867788
Sum of Squared Residuals      122.64462743
Regression F(1,96)                 35.9538
Significance Level of F          0.0000000
Log Likelihood                   -150.0478
Durbin-Watson Statistic             0.4395

    Variable                        Coeff      Std Error      T-Stat      Signif
**********************************************************************
```

```
1.  Constant                         10.20203661   0.23011125     44.33524  0.00000000
2.  TREND                            -0.02420111   0.00403611     -5.99615  0.00000004
```




```
set y = %resids
@acf(number=40) y
```

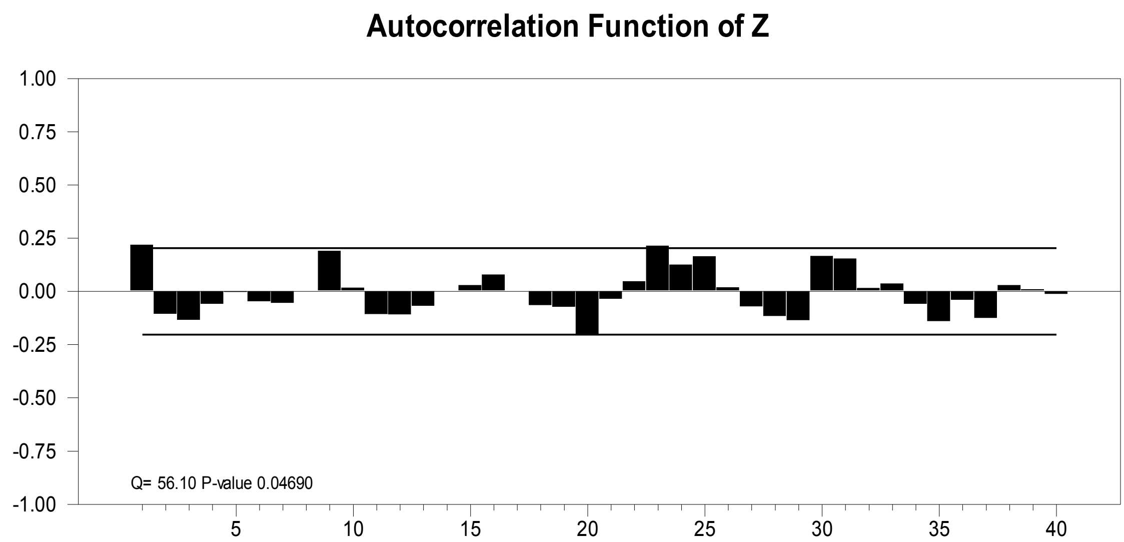
In a regression, or in a transformation instruction like SET, the notation series{lag} is used to denote the lag of a series.

```
linreg y
# y{1}
set z = %resids
```

```
Linear Regression - Estimation by Least Squares
Dependent Variable Y
Annual Data From 1876:01 To 1972:01
Usable Observations                     97
Degrees of Freedom                      96
Centered R^2                     0.6025021
R-Bar^2                          0.6025021
```

```
Uncentered R^2                       0.6025034
Mean of Dependent Variable       -0.002084170
Std Error of Dependent Variable   1.130096491
Standard Error of Estimate        0.712496882
Sum of Squared Residuals         48.734573399
Log Likelihood                      -104.2534
Durbin-Watson Statistic                1.5204

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  Y{1}                         0.7908423646 0.0655603471     12.06282  0.00000000
```

```
@acf(number=40) z
```



The instruction SCATTER is used for x-y graphs. (GRAPH is used when "time" is on the horizontal axis). SCATTER can't accept the lag notation used in the LINREG above, so we have to create ylag as a separate series. The LINE option on SCATTER is used to draw in a straight line. Use LINE=||intercept,slope||

```
set ylag = y{1}

scatter(footer="Figure 1-16 Scatter Plot of y(t-1) vs y(t)",$

  line=||0.0,%beta(1)||)

# ylag y
```

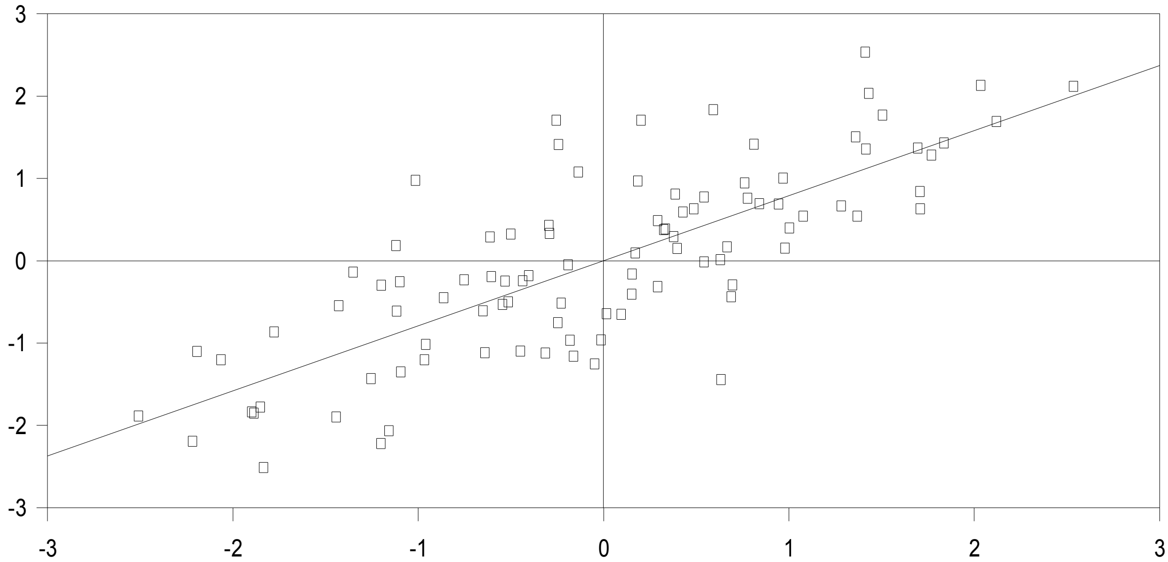


Figure 1-16 Scatter Plot of y(t-1) vs y(t)

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 1.5.1 from pp 25-26

*

open data strikes.dat

calendar 1951

data(format=free,org=columns) 1951:1 1980:1 strikes


print



 ENTRY         STRIKES

 1951:01         4737
```

1952:01	5117
1953:01	5091
1954:01	3468
1955:01	4320
1956:01	3825
1957:01	3673
1958:01	3694
1959:01	3708
1960:01	3333
1961:01	3367
1962:01	3614
1963:01	3362
1964:01	3655
1965:01	3963
1966:01	4405
1967:01	4595
1968:01	5045
1969:01	5700
1970:01	5716
1971:01	5138
1972:01	5010
1973:01	5353
1974:01	6074
1975:01	5031
1976:01	5648
1977:01	5506
1978:01	4230
1979:01	4827
1980:01	3885

The filter described in the text is done using the FILTER option with the options TYPE=CENTERED, WIDTH=5, and EXTEND=REPEAT. For a centered filter like this, the width is 2q+1 (the total number of terms).EXTEND=REPEAT requests the handling of the endpoints described(repeating the first and last values out-of-sample).

```
filter(type=centered,width=5,extend=repeat) strikes / sstrikes

graph(footer="Figure 1-18 Simple 5-term Moving Average of Strike Data",$

  overlay=dots,ovsame) 2

# sstrikes

# strikes
```

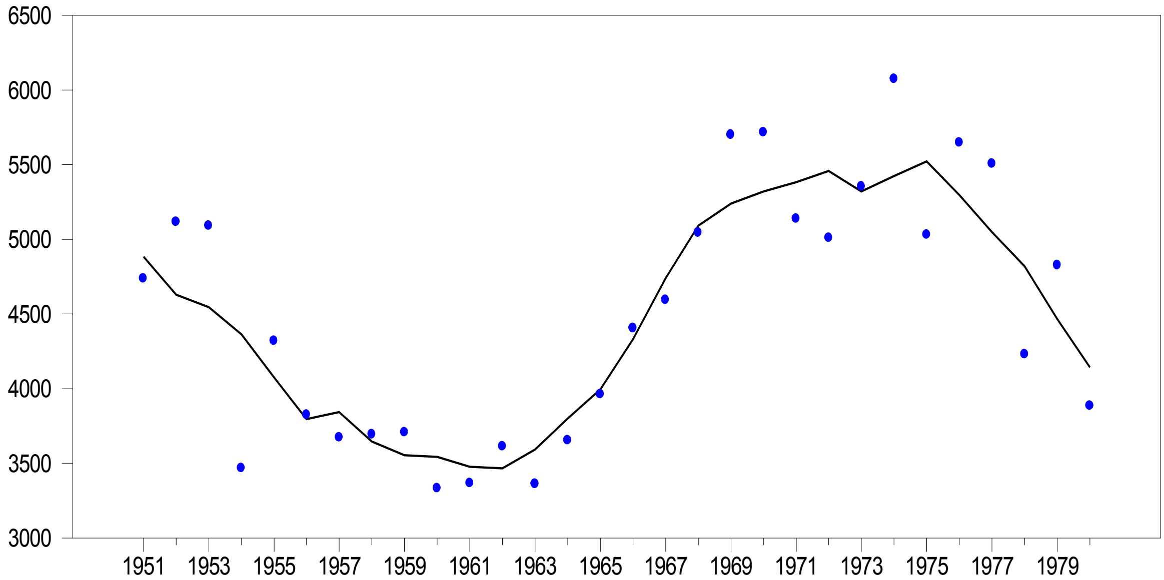


Figure 1-18 Simple 5-term Moving Average of Strike Data

```
set yhat = strikes-sstrikes

graph(footer="Figure 1-19 Residuals after Subtracting Moving Average")

# yhat
```

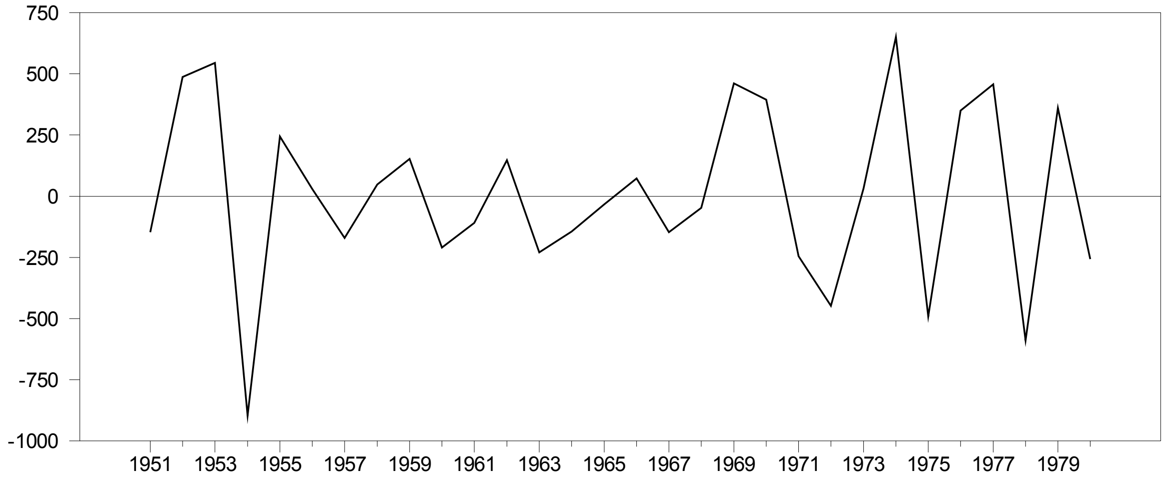


Figure 1-19 Residuals after Subtracting Moving Average

```
      Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 1.5.2 from pp 28-29


open data strikes.dat

calendar 1951

data(format=free,org=columns) 1951:1 1980:1 strikes

 @expsmooth(alpha=a) does exponential smoothing on the input series.

Note that there is a built-in instruction ESMOOTH which will do almost exactly this
calculation.

@expsmooth(alpha=.4) strikes / estrikes

graph(footer="Figure 1-21 Exponentially Smoothed Strike Data",$

   overlay=dots,ovsame) 2

# estrikes

# strikes
```

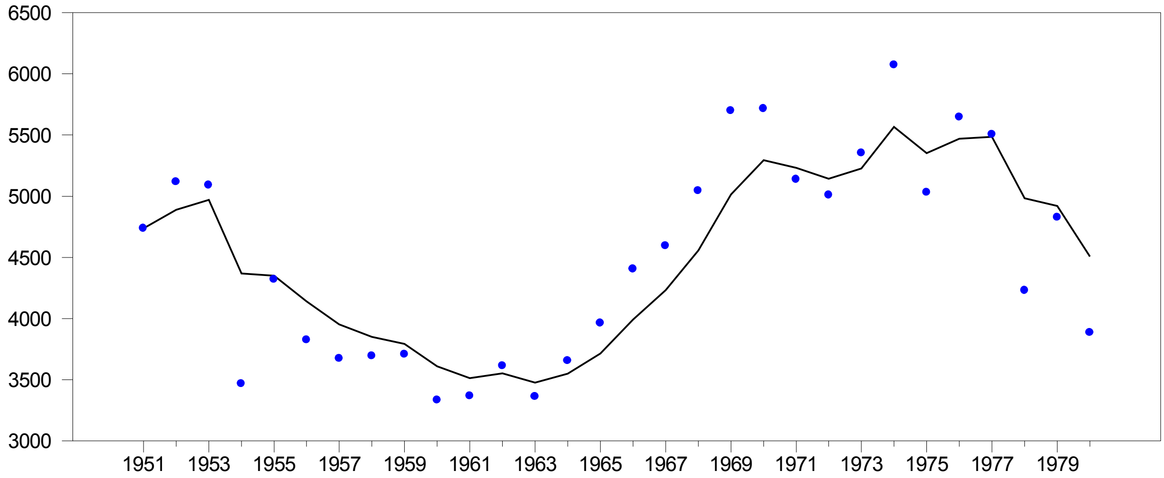


Figure 1-21 Exponentially Smoothed Strike Data

```
@freqsmooth(f=.4) strikes / fstrikes

graph(footer="Figure 1-22 Strike Data Smoothed by Elimination of High Frequencies",$

  overlay=dots,ovsame) 2

# fstrikes

# strikes
```

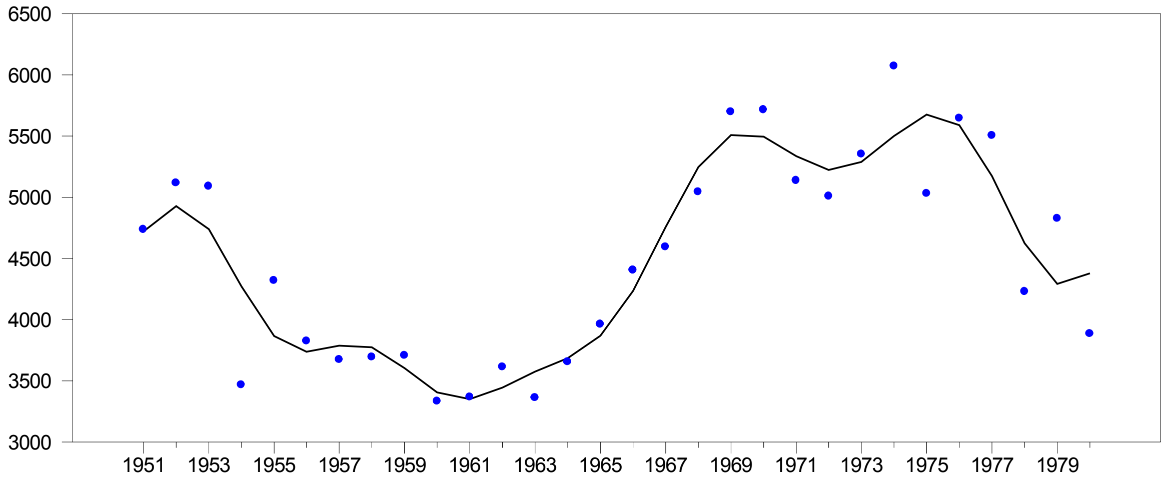


Figure 1-22 Strike Data Smoothed by Elimination of High Frequencies

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 1.5.3 from page 30
```

```
open data uspop.dat

calendar(years=10) 1790

data(format=free,org=columns) 1790:1 1990:1 uspop

to get a series differenced more than one, you can use the DIFFS

option to indicate the number of differencings needed.
```

```
diff(diffs=2) uspop / d2pop

graph(footer="Figure 1-23 Twice-Differenced U.S. Population")

# d2pop
```

Figure 1-23 Twice-Differenced U.S. Population

```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 1.5.4 from page 32

*

@ClassicalDecomp has options SPAN for the seasonal span (here not

* necessary, since the data have already been declared to be monthly),

* TREND=NONE/LINEAR/QUADRATIC to choose the type of trend, and FACTORS
```

```
* to get as a return the seasonal component.

@ClassicalDecomp(trend=quadratic,factors=s) deaths
set deseas = deaths-s

graph(footer="Figure 1-24 Deseasonalized Accidental Deaths")
# deseas
graph(footer="Figure 1-25 Seasonal Component")
# s
```

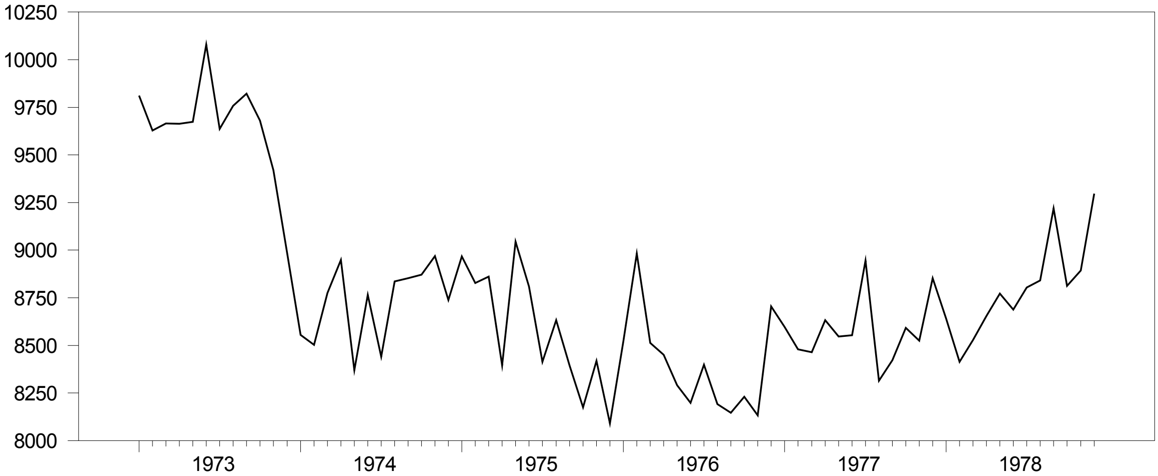


Figure 1-24 Deseasonalized Accidental Deaths

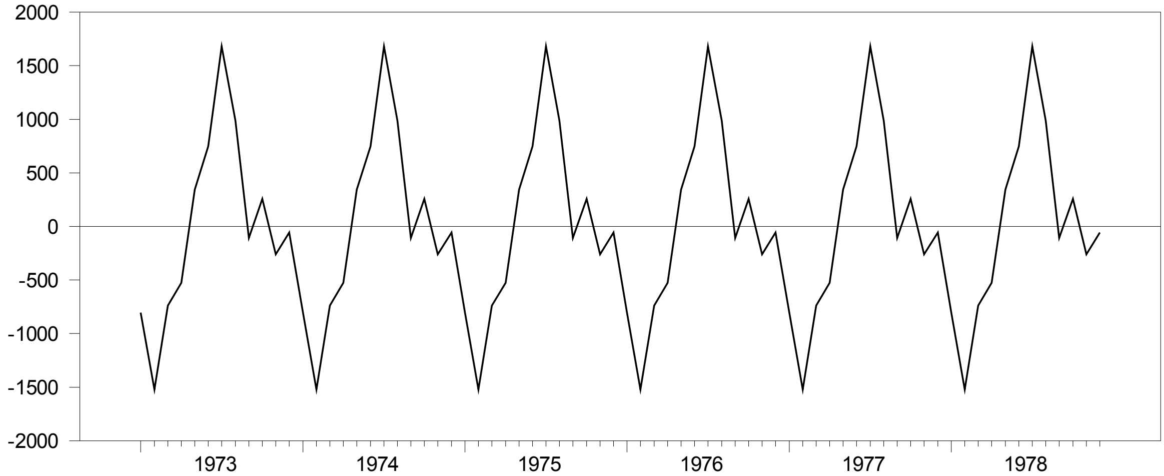


Figure 1-25 Seasonal Component

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 1.5.5 from pp 33-34


open data deaths.dat

calendar(m) 1973 1 12

data(format=free,org=columns) 1973:1 1978:12 deaths

You can do the seasonal difference with the DIFFERENCE instruction

* with the option SDIFFS=1 (1 seasonal difference). The span of the

* seasonal difference is taken from the CALENDAR under which you're

* currently working. For a non-consecutive differencing with some other

* gap, use SDIFFS=1 with SPAN=gap.


diff(sdiffs=1) deaths / d12

graph(footer="Figure 1-26 Seasonally Differenced Deaths")

# d12
```

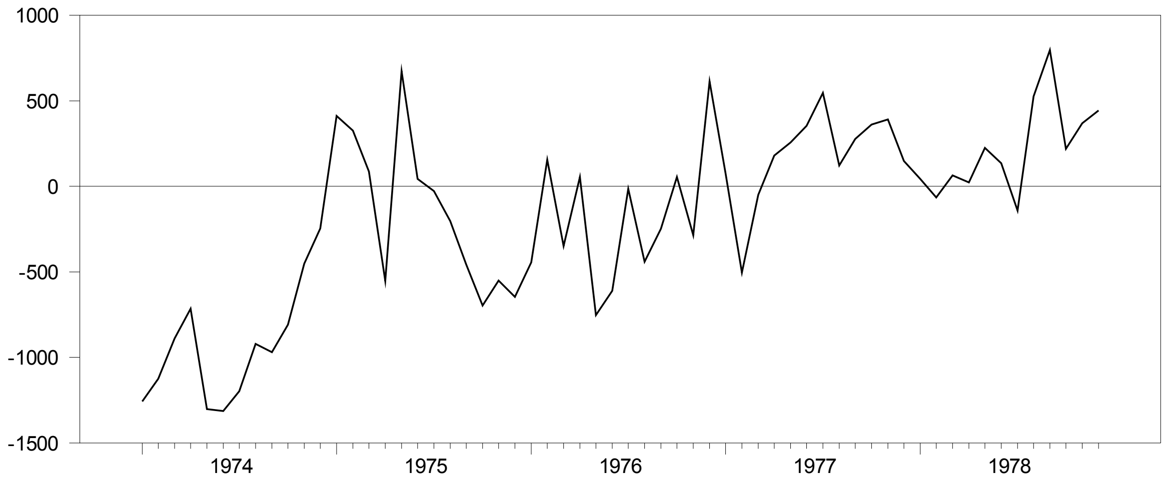


Figure 1-26 Seasonally Differenced Deaths

```
Difference the d12 series to get the series with the

* combined differencing.
```

```
diff d12 / d1_d12

graph(footer="Figure 1-27 Seasonally + Regular Differenced")

# d1_d12
```

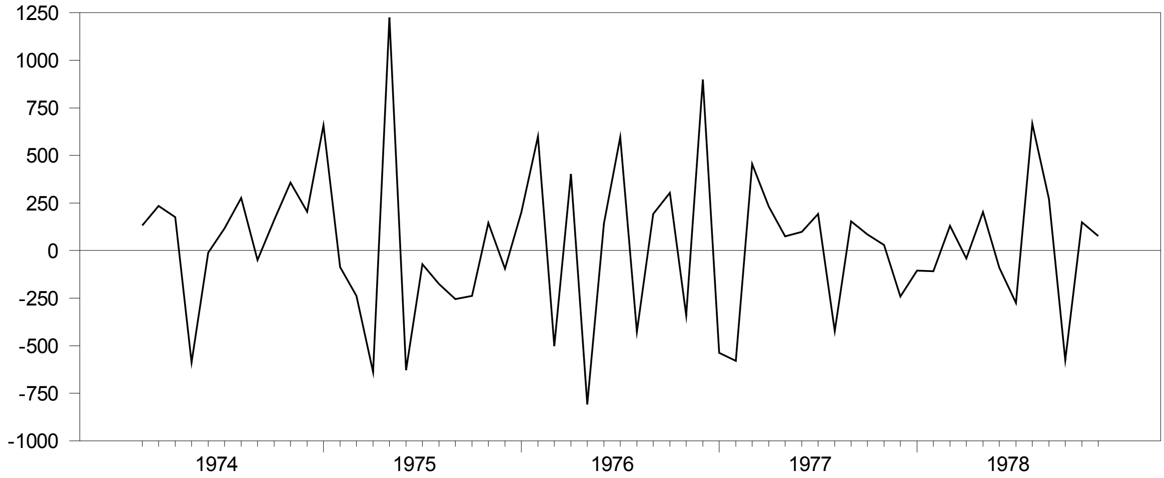


Figure 1-27 Seasonally + Regular Differenced

```
You can also do the two together on a single DIFFERENCE instruction -

* to do that, include both an SDIFFS and a DIFFS option.

diff(sdiffs=1,diffs=1) deaths / d1_d12

Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 1.6.1 from page 38

open data signal.dat

data(format=free,org=columns) 1 200 signal

     ENTRY       SIGNAL

      1      1.313713

      2      1.606903

      3      0.024068

      4      1.253616

      5      0.016990

      6     -0.608520

      7      0.055914

      8     -0.176190
```

9	0.890955
10	0.506149
11	-0.298280
12	0.928549
13	-0.746530
14	0.826843
15	0.729047
16	0.617480
17	1.288167
18	-0.870650
19	-0.325800
20	-0.154770
21	-2.304260
22	-1.382670
23	-0.316300
24	-0.149240
25	-0.141150
26	-2.110380
27	-1.742180
28	-0.116560
29	0.617320
30	0.896663
31	-1.969860
32	-0.220610
33	0.263709
34	-0.626550
35	-2.302920
36	-0.876590
37	0.732939

38	-0.274760
39	-1.564540
40	0.716348
41	-0.910940
42	-0.739790
43	-1.347260
44	0.900343
45	0.347532
46	-0.335040
47	-0.844080
48	-0.878340
49	-0.658920
50	-0.035900
51	-0.225250
52	1.221233
53	0.396642
54	0.252031
55	-0.790860
56	0.632941
57	0.176526
58	-0.085860
59	1.203176
60	1.601735
61	1.471174
62	0.890658
63	0.318089
64	-0.542840
65	-1.703500
66	0.694977

67	0.581209
68	0.255949
69	0.444875
70	-1.524010
71	-1.223690
72	1.579785
73	1.430638
74	-1.123540
75	0.507190
76	1.266679
77	-0.302490
78	-2.117990
79	2.161308
80	-0.649710
81	0.656040
82	1.824132
83	1.012981
84	-0.181670
85	0.667354
86	1.036289
87	-0.636940
88	0.400601
89	-0.266240
90	-1.148650
91	-0.281330
92	-1.044870
93	0.097074
94	-0.201040
95	-1.002470

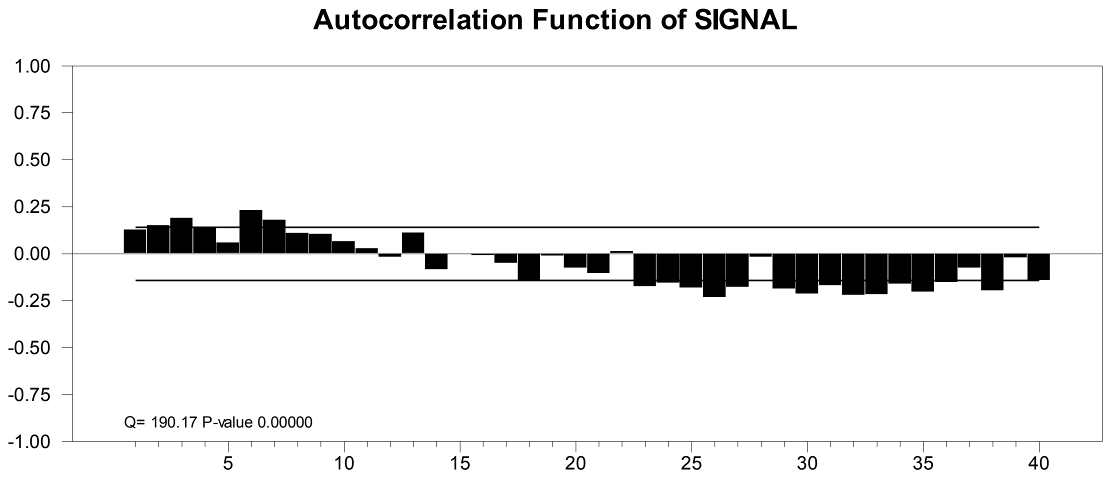
96	-0.453580
97	-2.145610
98	-2.363960
99	-1.090710
100	0.234067
101	-0.244850
102	-0.103940
103	0.798776
104	-0.113520
105	-0.527300
106	0.557788
107	-0.032090
108	0.780413
109	-0.850600
110	1.706892
111	-1.070740
112	-0.263790
113	1.439802
114	-0.957050
115	0.000547
116	0.662595
117	-1.291920
118	0.317573
119	1.664115
120	0.954180
121	0.775661
122	1.396079
123	-1.057670
124	0.368388

125	2.609669
126	0.083574
127	0.702958
128	0.635771
129	1.284480
130	-0.242080
131	1.890383
132	1.079300
133	0.713303
134	-0.747500
135	0.091361
136	-0.858790
137	0.167961
138	2.071657
139	1.130554
140	0.163430
141	0.585297
142	0.851185
143	-1.301590
144	-0.039940
145	-1.155480
146	-0.989930
147	0.870907
148	-0.812140
149	-0.240040
150	0.963996
151	-2.019430
152	-1.918750
153	-1.361980

154	-1.582420
155	-0.389460
156	-1.544560
157	-0.732730
158	-0.025300
159	-0.651590
160	-0.062270
161	-0.191710
162	-1.993630
163	0.410435
164	-1.716710
165	0.581560
166	-0.389410
167	-2.449540
168	0.025905
169	0.389097
170	-0.258060
171	0.138338
172	1.183930
173	-1.621800
174	-0.302160
175	-0.327200
176	0.643555
177	1.304706
178	0.680933
179	0.984866
180	-1.402110
181	1.255749
182	0.606834

```
    183      2.884288
    184     -0.391940
    185      2.299882
    186     -0.122790
    187      2.285628
    188      0.266665
    189      1.240692
    190     -0.218030
    191      0.830752
    192      0.185185
    193      1.435040
    194      1.937266
    195     -1.680960
    196      0.658784
    197     -0.132460
    198      0.279618
    199     -1.118010
    200      0.038985
```

```
@acf(number=40) signal
```



lags	IC
0	0.142
1	0.135
2	0.127
3	0.112
4	0.114
5	0.124
6	0.102
7	0.098*
8	0.107
9	0.117
10	0.125
11	0.134
12	0.136
13	0.143
14	0.135
15	0.144
16	0.154
17	0.162

```
18     0.162

19     0.172

20     0.182

21     0.191

22     0.194

23     0.191

24     0.196

25     0.201


@acf(number=40) signal

@bdindtests(number=20) signal

@yulelags(table) signal

Independence Tests for Series SIGNAL

Test            Statistic  P-Value

Ljung-Box Q(20) 55.4426305     0.0000

McLeod-Li(20)   17.9286225     0.5921

Turning Points   1.0108213     0.3121

Difference Sign  0.3665083     0.7140

Rank Test        0.7608511     0.4467


* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 3.2.8 from pages 96-98


open data oshorts.dat

data(format=free,org=columns) 1 57 overs


print
```

ENTRY	OVERS
1	78
2	-58
3	53
4	-65
5	13
6	-6
7	-16
8	-14
9	3
10	-72
11	89
12	-48
13	-14
14	32
15	56
16	-86
17	-66
18	50
19	26
20	59
21	-47
22	-83
23	2
24	-1
25	124
26	-106
27	113
28	-76

29	-47
30	-32
31	39
32	-30
33	6
34	-73
35	18
36	2
37	-24
38	23
39	-38
40	91
41	-56
42	-58
43	1
44	14
45	-4
46	77
47	-127
48	97
49	10
50	-28
51	-17
52	23
53	-2
54	48
55	-131
56	65
57	-17

```
graph(footer="Figure 3-5 Time series of the overshorts")
# overs
```

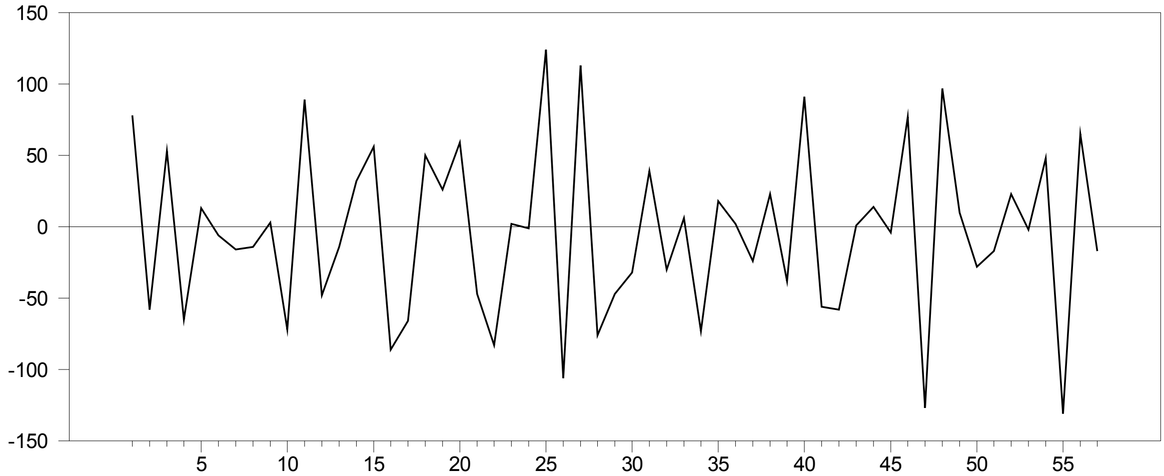
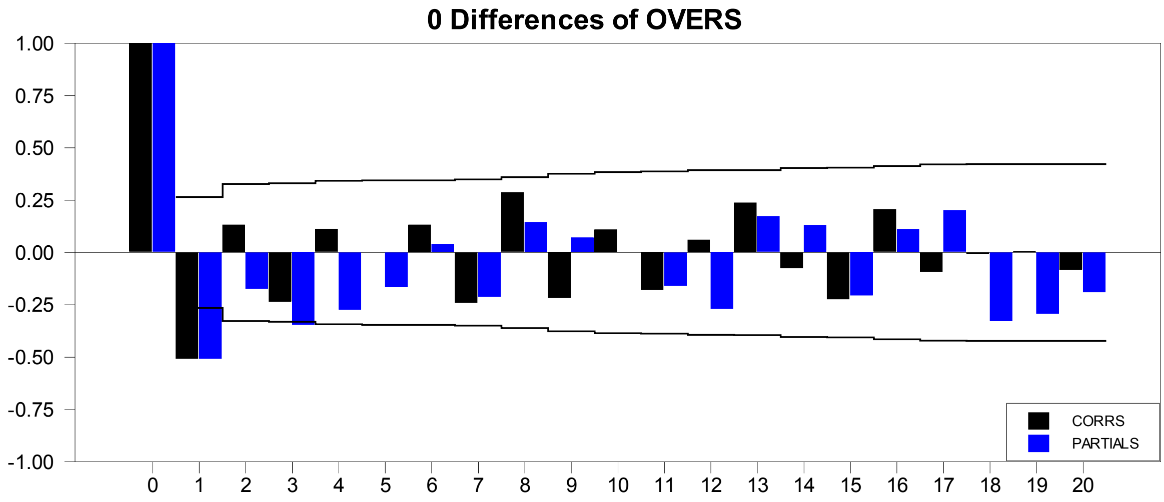


Figure 3-5 Time series of the overshorts

```
The two standard error bands in the BJIDENT output are computed using
* the extension of the formula used in Figure 3-6 to orders higher than
* one; that is, the limit shown from lag 1 is just 2/sqrt(n), but for
* lag 2, it's 2 sqrt(1+2r(1)^2)/sqrt(n) (which is the limit shown in
* figure 3-6) and 2 sqrt(1+2*(r(1)^2+r(2)^2))/sqrt(n) for lag 3, etc.

@bjident(number=20) overs
```




```
You can get the autocovariances by using the instruction CORRELATE

* with the option COVARIANCE. METHOD=YULE gives the same method of

* computing these as described in the book.


corr(covariance,number=4,method=yule) overs



Correlations of Series OVERS


Autocovariances

   0         1         2         3         4

 3415.72 -1719.96   416.70  -723.26   273.52


Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 3.2.9 on page 99

open data sunspots.dat

calendar 1770

data(format=free,org=columns) 1770:1 1869:1 sunspots
```

```
open data sunspots.dat

calendar 1770

data(format=free,org=columns) 1770:1 1869:1 sunspots


print
```

ENTRY	SUNSPOTS
1770:01	101
1771:01	82
1772:01	66
1773:01	35
1774:01	31
1775:01	7
1776:01	20
1777:01	92
1778:01	154
1779:01	125
1780:01	85
1781:01	68
1782:01	38
1783:01	23
1784:01	10
1785:01	24
1786:01	83
1787:01	132
1788:01	131
1789:01	118
1790:01	90

1791:01	67
1792:01	60
1793:01	47
1794:01	41
1795:01	21
1796:01	16
1797:01	6
1798:01	4
1799:01	7
1800:01	14
1801:01	34
1802:01	45
1803:01	43
1804:01	48
1805:01	42
1806:01	28
1807:01	10
1808:01	8
1809:01	2
1810:01	0
1811:01	1
1812:01	5
1813:01	12
1814:01	14
1815:01	35
1816:01	46
1817:01	41
1818:01	30
1819:01	24

1820:01	16
1821:01	7
1822:01	4
1823:01	2
1824:01	8
1825:01	17
1826:01	36
1827:01	50
1828:01	62
1829:01	67
1830:01	71
1831:01	48
1832:01	28
1833:01	8
1834:01	13
1835:01	57
1836:01	122
1837:01	138
1838:01	103
1839:01	86
1840:01	63
1841:01	37
1842:01	24
1843:01	11
1844:01	15
1845:01	40
1846:01	62
1847:01	98
1848:01	124

```
 1849:01          96
 1850:01          66
 1851:01          64
 1852:01          54
 1853:01          39
 1854:01          21
 1855:01           7
 1856:01           4
 1857:01          23
 1858:01          55
 1859:01          94
 1860:01          96
 1861:01          77
 1862:01          59
 1863:01          44
 1864:01          47
 1865:01          30
 1866:01          16
 1867:01           7
 1868:01          37
 1869:01          74

@bjident(number=40) sunspots
You can get the autocovariances by using the instruction CORRELATE
* with the option COVARIANCE. METHOD=YULE gives the same method of
* computing these as described in the book.
corr(covariance,number=4,method=yule) sunspots


corr(covariance,number=4,method=yule) sunspots
```

```
Correlations of Series SUNSPOTS

Annual Data From 1770:01 To 1869:01


Autocovariances

   0        1        2        3        4

 1382.19  1114.38   591.72    96.22  -234.17
```

```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 4.1.5 from pp 119-122


This uses the ARMASpectrum procedure, which takes an input ARMA model

* and produces a graph of its spectral density.


equation(noconst,coeffs=||.7||) ar1 y 1 0

@ARMASpectrum(footer="Figure 4-3") ar1

equation(noconst,coeffs=||-.7||) ar1 y 1 0

@ARMASpectrum(footer="Figure 4-4") ar1

equation(noconst,coeffs=||.9||) ma1 y 0 1

@ARMASpectrum(footer="Figure 4-5") ma1

equation(noconst,coeffs=||-.9||) ma1 y 0 1

@ARMASpectrum(footer="Figure 4-6") ma1
```

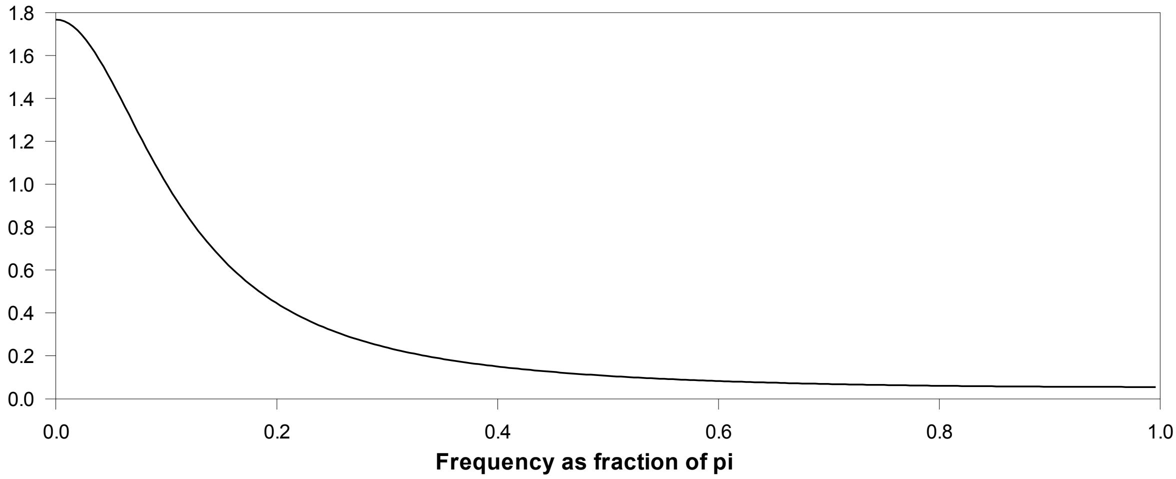


Figure 4-3

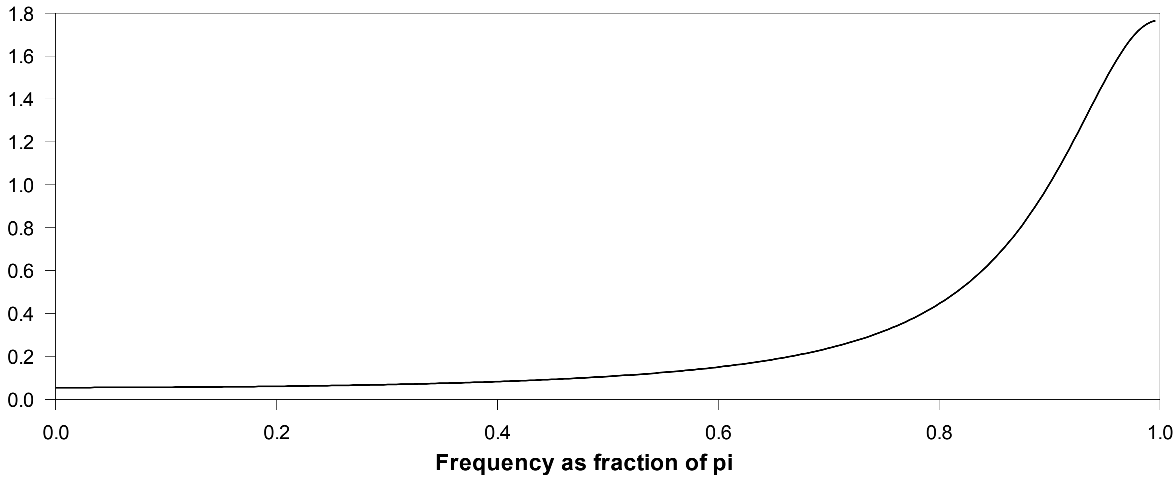


Figure 4-4

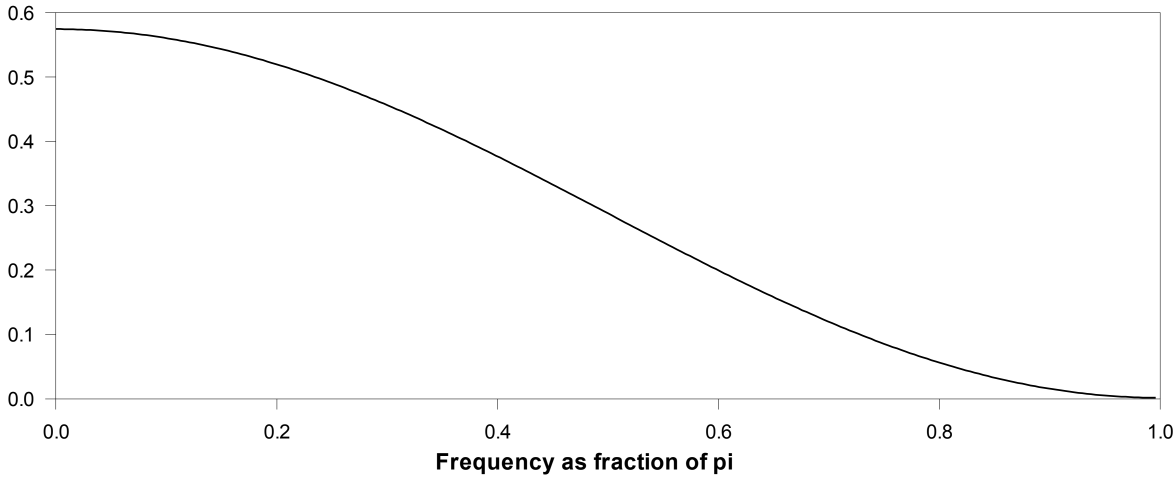


Figure 4-5

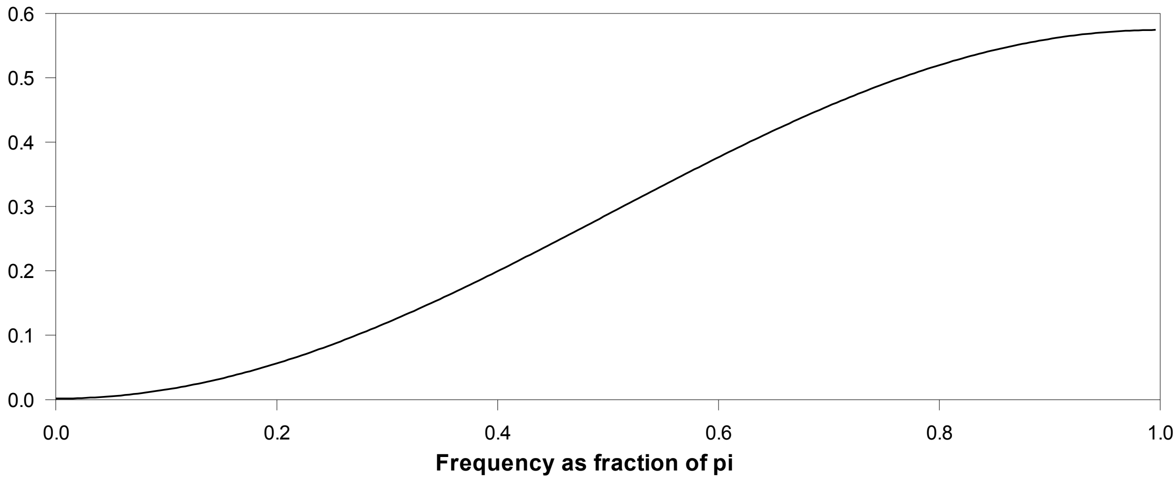


Figure 4-6

```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 4.2.2 on page 127. (Graphs are figures 4-9 to 4-11).


cal 1770

open data sunspots.dat

data 1770:1 1869:1 spots

stats spots
```

```
@BDSpectrum is a simplified version of the RATS procedure @Spectrum.
* It has only the HEADER and FOOTER (for the graph), WIDTH (for flat
* smoothing of the periodogram) and WEIGHTS (for weighted smoothing).

@BDSpectrum(footer="Figure 4-9 Periodogram") spots
@BDSpectrum(footer="Figure 4-10 Lightly smoothed estimate",width=1) spots
@BDSpectrum(footer="Figure 4-11 Double smoothed estimate",$
  weights=||3.0,3.0,2.0,1.0||) spots

* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 4.4.1 from pp 133-134
*
open data sunspots.dat
cal 1770
data 1770:1 1869:1 spots
boxjenk(ar=2,demean,maxl,method=init,define=ar2) spots
@armaspectrum(footer="Figure 4-14. Spectral Density of Sunspot Data") ar2

Box-Jenkins - Estimation by Initial Guess Algorithm
Dependent Variable SPOTS
Annual Data From 1770:01 To 1869:01
Usable Observations                       100
Degrees of Freedom                         98
Centered R^2                        0.8315023
R-Bar^2                             0.8297830
Uncentered R^2                      0.9350292
Mean of Dependent Variable       46.930000000
Std Error of Dependent Variable  37.365044703
Standard Error of Estimate       15.415834031
```

```
Sum of Squared Residuals         23289.498011

Log Likelihood                      -415.4625

Durbin-Watson Statistic                1.5501

Q(25-2)                               25.8385

Significance Level of Q             0.3085471


    Variable                        Coeff
**********************************************
1.  AR{1}                         1.317500535
2.  AR{2}                        -0.634121487
```

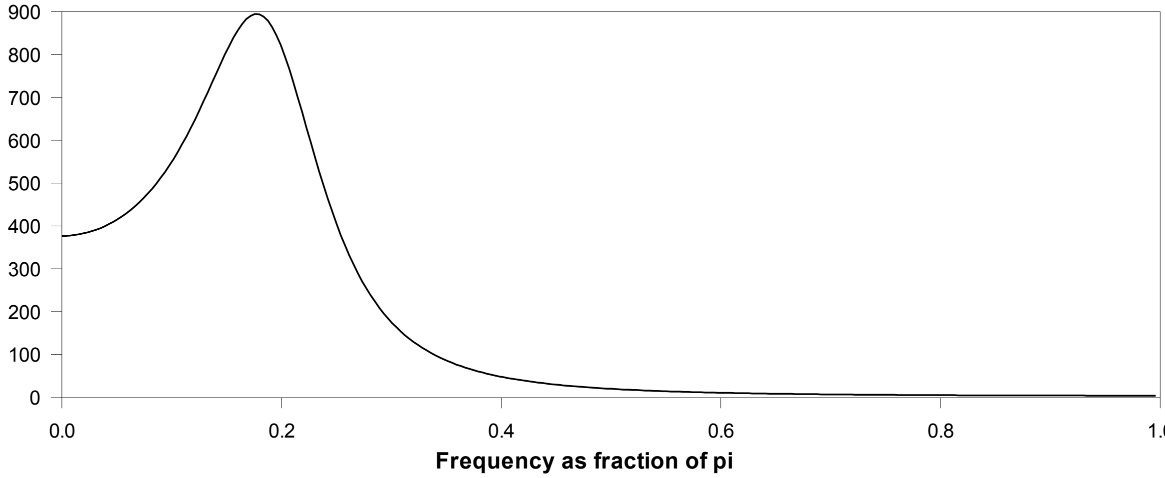


Figure 4-14. Spectral Density of Sunspot Data

```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 5.1.1 from pp 143-144

open data dowj.dat
data(format=free,org=columns) 1 78 dowj
```

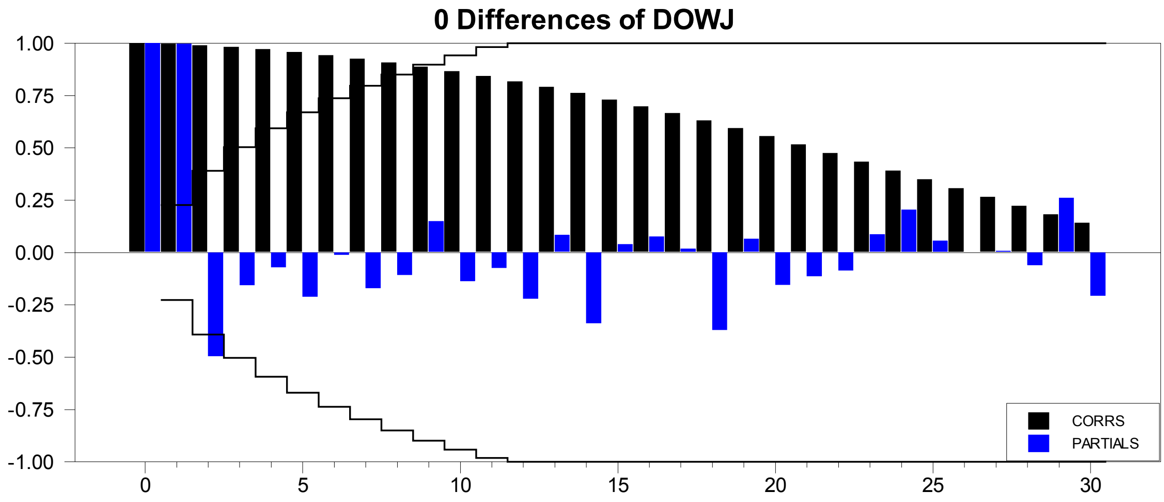
```
print
```

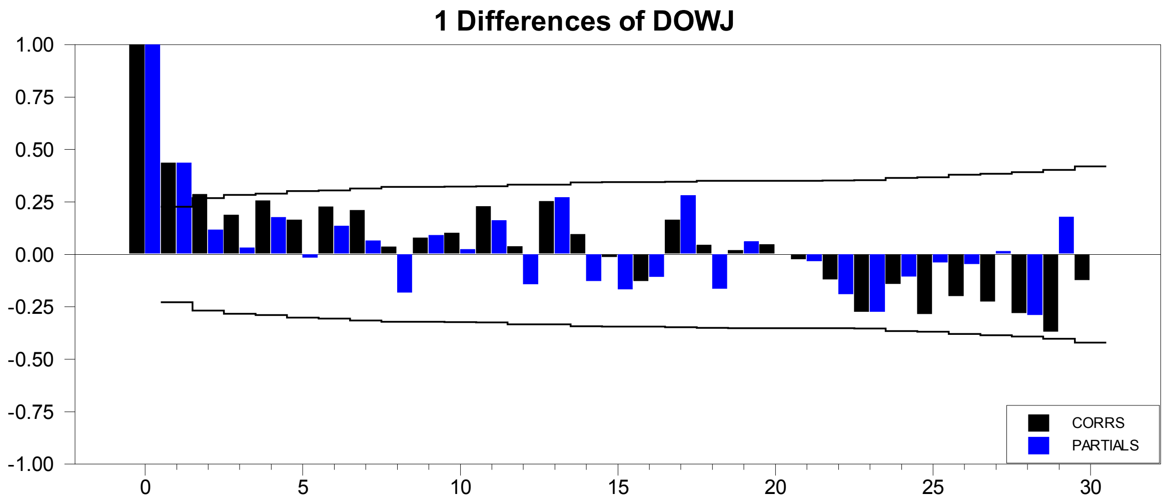
ENTRY	DOWJ
1	110.94
2	110.69
3	110.43
4	110.56
5	110.75
6	110.84
7	110.46
8	110.56
9	110.46
10	110.05
11	109.60
12	109.31
13	109.31
14	109.25
15	109.02
16	108.54
17	108.77
18	109.02
19	109.44
20	109.38
21	109.53
22	109.89
23	110.56
24	110.56

25	110.72
26	111.23
27	111.48
28	111.58
29	111.90
30	112.19
31	112.06
32	111.96
33	111.68
34	111.36
35	111.42
36	112.00
37	112.22
38	112.70
39	113.15
40	114.36
41	114.65
42	115.06
43	115.86
44	116.40
45	116.44
46	116.88
47	118.07
48	118.51
49	119.28
50	119.79
51	119.70
52	119.28
53	119.66

54	120.14
55	120.97
56	121.13
57	121.55
58	121.96
59	122.26
60	123.79
61	124.11
62	124.14
63	123.37
64	123.02
65	122.86
66	123.02
67	123.11
68	123.05
69	123.05
70	122.83
71	123.18
72	122.67
73	122.73
74	122.86
75	122.67
76	122.09
77	122.00
78	121.23

```
@bjident(diffs=1,number=30) dowj
```






```
set ddow = dowj-dowj{1}

This does the Durbin-Levinson recursion for up to 3 lags

@durbinlevinson(m=3,phi=phi,v=v) ddow

The matrix phi has the estimated autoregressions in the rows
*disp "Durbin-Levinson Recursion Matrix"
disp phi
```

Durbin-Levinson Recursion Matrix

0.42188	0.00000	0.00000
0.37388	0.11378	0.00000
0.37217	0.10817	0.01500

```
@arautolags(crit=caic,method=yule,max=27,table) ddow
```

AIC-Corrected Lag Analysis for DDOW

Lags	IC
0	-1.689
1	-1.858*
2	-1.843
3	-1.815
4	-1.812
5	-1.782
6	-1.764
7	-1.734
8	-1.725
9	-1.693
10	-1.658
11	-1.643
12	-1.624
13	-1.636
14	-1.606
15	-1.609
16	-1.579
17	-1.578
18	-1.537
19	-1.491

```
  20    -1.450
  21    -1.400
  22    -1.354
  23    -1.346
  24    -1.290
  25    -1.234
  26    -1.183
  27    -1.120
```

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 5.1.5 from pp 153-154
*
open data dowj.dat
data(format=free,org=columns) 1 78 dowj
*
set ddow = dowj-dowj{1}
*
@maautolags(maxlag=17,crit=bic,table) ddow
boxjenk(ma=1,maxl,demean) ddow
boxjenk(ma=2,maxl,demean) ddow
boxjenk(ma=3,maxl,demean) ddow
@maautolags(maxlag=17,crit=bic,table) ddow
boxjenk(ma=1,maxl,demean) ddow
boxjenk(ma=2,maxl,demean) ddow
boxjenk(ma=3,maxl,demean) ddow
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in    17 Iterations. Final criterion was  0.0000080 <=  0.0000100
```

```
Dependent Variable DDOW
Usable Observations                     77
Degrees of Freedom                      76
Centered R^2                     0.1394629
R-Bar^2                          0.1394629
Uncentered R^2                   0.2171665
Mean of Dependent Variable    0.1336363636
Std Error of Dependent Variable  0.4269503518
Standard Error of Estimate    0.3960610790
Sum of Squared Residuals      11.921692751
Log Likelihood                    -37.4968
Durbin-Watson Statistic             1.7444
Q(19-1)                            34.5083
Significance Level of Q          0.0108927

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  MA{1}                        0.3314819153 0.1127366088      2.94032  0.00434208



Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in    15 Iterations. Final criterion was  0.0000052 <=  0.0000100
Dependent Variable DDOW
Usable Observations                     77
Degrees of Freedom                      75
Centered R^2                     0.1873093
R-Bar^2                          0.1764734
Uncentered R^2                   0.2606925
Mean of Dependent Variable    0.1336363636
```

Std Error of Dependent Variable	0.4269503518
Standard Error of Estimate	0.3874504612
Sum of Squared Residuals	11.258839493
Log Likelihood	-35.3418
Durbin-Watson Statistic	1.9173
Q(19-2)	31.0802
Significance Level of Q	0.0195282

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	0.4109803511	0.1161228688	3.53919	0.00069317
2. MA{2}	0.2077207988	0.1168024919	1.77839	0.07939196

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in 16 Iterations. Final criterion was 0.0000078 <= 0.0000100

Dependent Variable DDOW

Usable Observations	77
Degrees of Freedom	74
Centered R^2	0.1877164
R-Bar^2	0.1657628
Uncentered R^2	0.2610629
Mean of Dependent Variable	0.1336363636
Std Error of Dependent Variable	0.4269503518
Standard Error of Estimate	0.3899618776
Sum of Squared Residuals	11.253199681
Log Likelihood	-35.3188
Durbin-Watson Statistic	1.9182
Q(19-3)	31.1573

Significance Level of Q 0.0128470

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	0.4043897252	0.1195040073	3.38390	0.00114568
2. MA{2}	0.2155523094	0.1253246166	1.71995	0.08962105
3. MA{3}	0.0274248362	0.1206926598	0.22723	0.82087251

lags	IC
0	-1.659
1	-1.798*
2	-1.755
3	-1.699
4	-1.669
5	-1.613
6	-1.569
7	-1.515
8	-1.482
9	-1.427
10	-1.371
11	-1.336
12	-1.297
13	-1.291
14	-1.244
15	-1.231
16	-1.186
17	-1.172

Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

```
* Example 5.1.7 from page 157
*
* ITSM uses 20+p+q lags on the 1st stage in Hannan-Rissanen
*
@HannanRissanen(ar=1,ma=1,m=22,nocorrect) lake
```

```
Equation Estimated by Hannan-Rissanen

Dependent Variable XC
     Variable            Coeff
**********************************
1.   XC{1}            0.6960771500
2.   Mvg Avge{1}      0.3787969217
```

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 5.1.7 from page 157
open data lake.dat
calendar 1875
data(format=free,org=columns) 1875:1 1972:1 lake
* ITSM uses 20+p+q lags on the 1st stage in Hannan-Rissanen
*
@HannanRissanen(ar=1,ma=1,m=22,nocorrect) lake
```

```
Equation Estimated by Hannan-Rissanen

Dependent Variable XC
     Variable            Coeff
**********************************
```

```
1.   XC{1}            0.6960771500
2.   Mvg Avge{1}      0.3787969217
```

Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

```
* Example 5.2.4

* This estimates the model by maximum likelihood, after extracting the
* sample mean.
*
boxjenk(ar=1,demean,maxl) ddow
boxjenk(ar=1,diffs=1,demean,maxl) dowj
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     1 Iterations. Final criterion was  0.0000000 <=  0.0000100
Dependent Variable DOWJ
Usable Observations                       77
Degrees of Freedom                        76
Centered R^2                       0.9940068
R-Bar^2                            0.9940068
Uncentered R^2                     0.9999866
Mean of Dependent Variable      115.74493506
Std Error of Dependent Variable   5.51504655
Standard Error of Estimate        0.42695035
Sum of Squared Residuals        13.853781818
Log Likelihood                      -43.2213
Durbin-Watson Statistic               1.0867
Q(19-1)                              46.2524
Significance Level of Q            0.0002725
```

```
    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                        0.8000000000 0.0000000000      0.00000  0.00000000*
This will produce slightly different results by estimating the mean as
* part of the model. (The estimates of the mean differ because the error
* variance for the full model is slightly higher for the first few data
* points, so they won't get equal weight in estimating the mean).
*
boxjenk(ar=1,constant,maxl) ddow
*
* This is an alternative to the original set of estimates. Instead of
* transforming the series to differences separately, you can include a
* DIFFS option on the BOXJENK and apply the instruction to the original
* series. Note that the "de-meaning" takes place after the data have
* been differenced
*
boxjenk(ar=1,diffs=1,demean,maxl) dowj
```


```
boxjenk(ar=1,diffs=1,demean,maxl) dowj
```


```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     1 Iterations. Final criterion was  0.0000000 <=  0.0000100
Dependent Variable DOWJ
Usable Observations                        77
Degrees of Freedom                         76
Centered R^2                        0.9940068
R-Bar^2                             0.9940068
Uncentered R^2                      0.9999866
Mean of Dependent Variable       115.74493506
Std Error of Dependent Variable    5.51504655
```

```
Standard Error of Estimate         0.42695035
Sum of Squared Residuals         13.853781818
Log Likelihood                       -43.2213
Durbin-Watson Statistic                1.0867
Q(19-1)                               46.2524
Significance Level of Q             0.0002725

    Variable                       Coeff      Std Error      T-Stat      Signif
*************************************************************************************
1.  AR{1}                       0.8000000000 0.0000000000      0.00000  0.00000000
```

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 6.1.1 from pp 181-185
open data ex611.dat
data(format=free,org=columns) 1 200 x
*
print
```

ENTRY	X
1	2.35529
2	3.97812
3	4.68998
4	4.76872
5	4.51269
6	4.46574
7	3.15743
8	2.80433
9	3.12643
10	2.23670

11	1.72745
12	0.60372
13	-0.07420
14	-1.59202
15	-2.45998
16	-2.01945
17	-1.74457
18	-1.84849
19	-3.04644
20	-4.41689
21	-6.11269
22	-7.85280
23	-10.21754
24	-12.17186
25	-14.19516
26	-13.36829
27	-12.66769
28	-11.86291
29	-9.79885
30	-7.15658
31	-6.78851
32	-7.23582
33	-7.22482
34	-7.51158
35	-7.94588
36	-6.52517
37	-6.20661
38	-5.36547
39	-5.48722

40	-4.41160
41	-3.56551
42	-2.38470
43	-0.67871
44	1.82081
45	4.98448
46	7.90982
47	10.95520
48	14.66206
49	18.21799
50	20.21167
51	20.72451
52	21.38262
53	21.46560
54	22.62539
55	23.38983
56	23.37781
57	22.73517
58	22.32486
59	21.39278
60	20.42269
61	20.17254
62	20.10231
63	21.85471
64	24.78262
65	26.75943
66	27.80584
67	26.67552
68	24.99831

69	25.40102
70	26.32479
71	29.55460
72	31.43178
73	32.87692
74	34.16300
75	36.58532
76	38.28238
77	39.30827
78	41.30084
79	43.14825
80	44.14237
81	43.80638
82	43.32075
83	43.96524
84	42.95986
85	40.63035
86	39.79926
87	39.42160
88	38.17426
89	35.54543
90	33.16446
91	29.67088
92	26.68562
93	25.22821
94	25.61048
95	27.41360
96	28.72619
97	27.50701

98	25.34407
99	24.31539
100	23.86783
101	25.07933
102	27.61560
103	29.41568
104	29.46099
105	28.39221
106	28.82964
107	28.23829
108	26.42978
109	26.11173
110	25.85741
111	26.38761
112	27.14765
113	27.12752
114	28.42725
115	29.90639
116	31.92018
117	33.79117
118	35.27895
119	36.05540
120	35.71111
121	35.31327
122	36.32458
123	38.04168
124	40.49825
125	43.24348
126	44.53748

127	44.59506
128	44.66600
129	45.54657
130	46.40500
131	47.16366
132	46.92207
133	46.63551
134	48.39925
135	48.22291
136	49.83832
137	51.86087
138	53.01292
139	55.01277
140	57.05932
141	60.92378
142	65.11794
143	69.95896
144	72.87776
145	76.03131
146	78.92613
147	79.52071
148	80.56388
149	82.47683
150	83.23601
151	83.52681
152	82.63753
153	82.21608
154	82.26743
155	82.92201

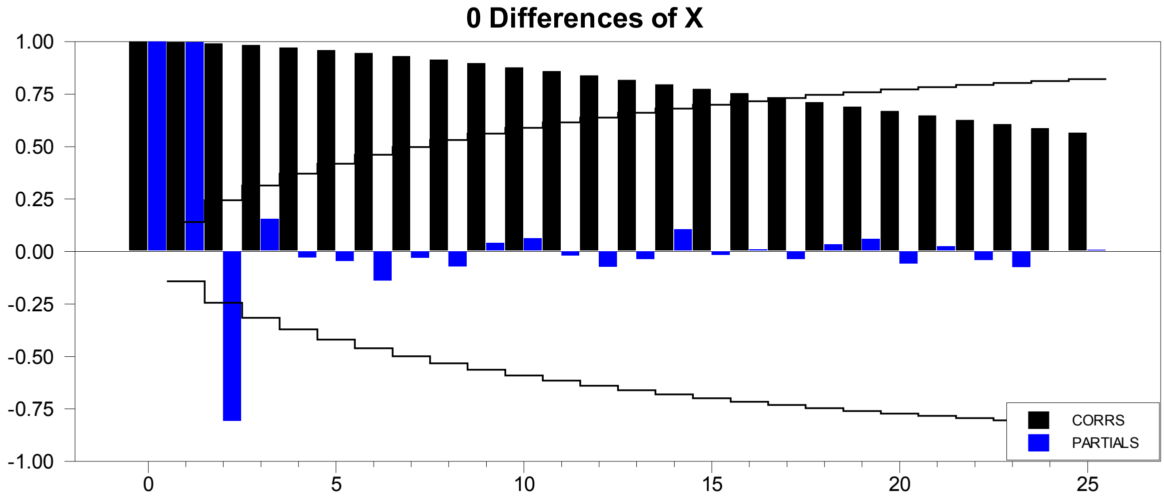
156	83.90839
157	84.05912
158	83.55289
159	81.31308
160	76.75011
161	70.99807
162	66.86860
163	64.33958
164	61.98777
165	59.70756
166	57.68674
167	56.43822
168	55.89503
169	54.62747
170	54.38974
171	52.59788
172	50.08789
173	47.04164
174	45.29583
175	44.26649
176	44.43901
177	43.77127
178	42.40905
179	41.81829
180	40.24721
181	39.22478
182	38.72275
183	37.55967
184	36.65595

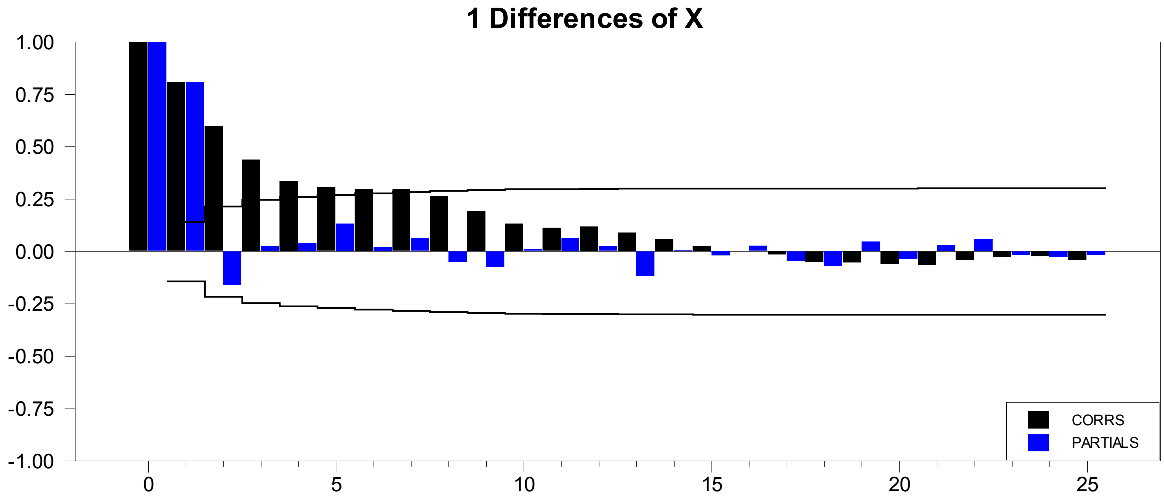
185	34.95921
186	32.29345
187	32.41226
188	33.11725
189	33.77816
190	33.56976
191	32.78857
192	33.88009
193	34.96359
194	34.75066
195	34.13862
196	31.96639
197	30.02671
198	29.22991
199	27.89676
200	26.96949

```
graph(footer="Figure 6.1")
# x
```

Figure 6.1

```
JIDENT with DIFFS=1 provides correlograms of the levels and first

* differenced data.


@bjident(diffs=1) x
```






```
Estimate ARIMA(1,1,0) model

boxjenk(diffs=1,ar=1,maxl) x



Box-Jenkins - Estimation by ML Gauss-Newton
```

```
Convergence in     1 Iterations. Final criterion was  0.0000000 <=  0.0000100
Dependent Variable X
Usable Observations                        199
Degrees of Freedom                         198
Centered R^2                         0.9953447
R-Bar^2                              0.9953447
Uncentered R^2                       0.9982162
Mean of Dependent Variable        31.299755126
Std Error of Dependent Variable   24.731988134
Standard Error of Estimate         1.687465273
Sum of Squared Residuals          563.81273147
Log Likelihood                       -385.9898
Durbin-Watson Statistic                 0.3802
Q(36-1)                               385.9946
Significance Level of Q              0.0000000

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                        0.8000000000 0.0000000000      0.00000  0.00000000

Estimate AR(2) model
*
boxjenk(ar=2,maxl) x
boxjenk(ar=2,maxl) x



Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     7 Iterations. Final criterion was  0.0000065 <=  0.0000100
Dependent Variable X
```

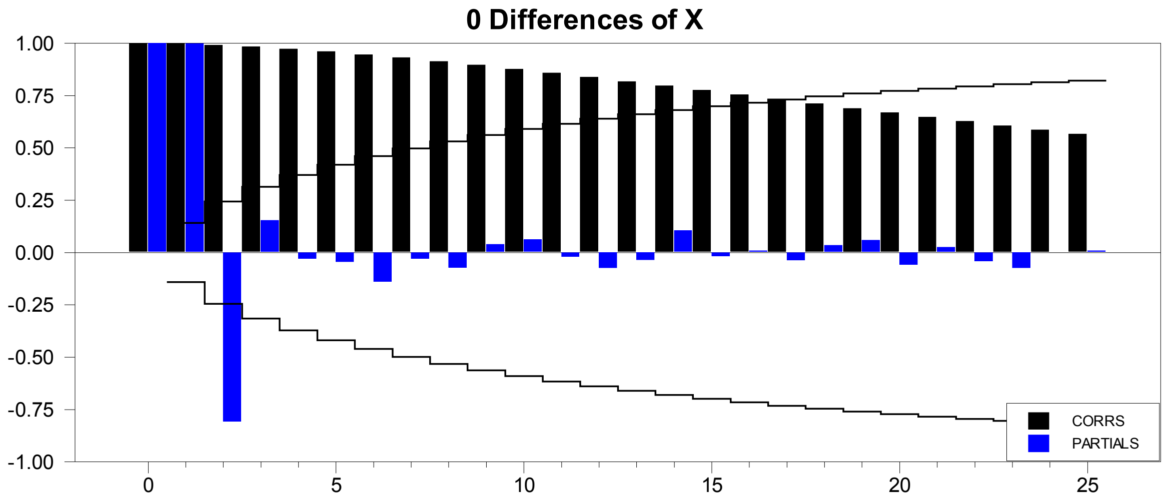
```
Usable Observations                          200
Degrees of Freedom                           198
Centered R^2                           0.9984090
R-Bar^2                                0.9984009
Uncentered R^2                         0.9993862
Mean of Dependent Variable          31.155032800
Std Error of Dependent Variable     24.754523274
Standard Error of Estimate           0.989887975
Sum of Squared Residuals           194.01588402
Log Likelihood                         -284.8147
Durbin-Watson Statistic                   1.7372
Q(36-2)                                  28.3821
Significance Level of Q                0.7391619

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                         1.803848140  0.040708571     44.31126  0.00000000
2.  AR{2}                        -0.806017957  0.040758099    -19.77565  0.00000000
```

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 6.2.2 from page 193
*
boxjenk(ar=1,ma=1,demean,maxl) lake
@bdindtests %resids
diff(center) lake / clake
@bjautofit(pmax=5,qmax=5) clake
```

Box-Jenkins - Estimation by ML Gauss-Newton

```
Convergence in     4 Iterations. Final criterion was  0.0000007 <=  0.0000100

Dependent Variable LAKE

Annual Data From 1875:01 To 1972:01

Usable Observations                        98

Degrees of Freedom                         96

Centered R^2                        0.7238400

R-Bar^2                             0.7209633

Uncentered R^2                      0.9942623

Mean of Dependent Variable       9.0040816327

Std Error of Dependent Variable  1.3182985260

Standard Error of Estimate       0.6963770016

Sum of Squared Residuals         46.554329119

Log Likelihood                      -103.2561

Durbin-Watson Statistic                1.9786

Q(24-2)                               13.3998

Significance Level of Q             0.9214065


    Variable                        Coeff      Std Error      T-Stat      Signif

************************************************************************************

1.  AR{1}                        0.7445704576 0.0778787076      9.56064  0.00000000

2.  MA{1}                        0.3212836124 0.1129605392      2.84421  0.00544119



Independence Tests for Series %RESIDS

Test            Statistic  P-Value

Ljung-Box Q(19)   8.607445     0.9794

McLeod-Li(19)    21.320948     0.3193

Turning Points    1.209127     0.2266

Difference Sign   0.522233     0.6015
```

```
Rank Test        -1.801679     0.0716



AIC analysis of models for series CLAKE

   MA

AR   0        1         2        3        4        5

 0 331.2698 251.2965  226.9329 218.1268 218.5133 218.6944

 1 333.2698 210.5121* 212.4842 213.9336 215.3888 217.3678

 2 211.2834 212.4967  214.0806 215.5736 217.3667 218.2391

 3 212.0670 213.4877  215.4874 217.4830 215.6806 217.3243

 4 213.6667 215.4873  217.1059 219.4830 220.1542 224.4347

 5 215.6087 216.3504  218.2990 219.7162 219.6792 262.7538


* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 6.3.1 from pp 195-196


open data ex611.dat

data(format=free,org=columns) 1 200 x


@bjident x

set dx = x-x{1}
```




```
linreg dx
# constant x{1} dx{1 2}
```

```
Linear Regression - Estimation by Least Squares
Dependent Variable DX
Usable Observations                     197
Degrees of Freedom                      193
Centered R^2                      0.6666930
R-Bar^2                           0.6615121
Uncentered R^2                    0.6681909
Mean of Dependent Variable     0.1130939594
Std Error of Dependent Variable 1.6875183016
Standard Error of Estimate     0.9817932974
Sum of Squared Residuals       186.03618921
Regression F(3,193)                128.6819
Significance Level of F           0.0000000
Log Likelihood                    -273.8905
```

```
Durbin-Watson Statistic                1.9923


    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  Constant                     0.150279326  0.113477590      1.32431  0.18696677
2.  X{1}                        -0.004113559  0.002839488     -1.44870  0.14904485
3.  DX{1}                        0.933549178  0.070748894     13.19525  0.00000000
4.  DX{2}                       -0.154797403  0.070790303     -2.18670  0.02996668
```

```
* This will have a slightly different value, because the calculation is being done to
a higher precision than the one shown in the text, which uses only four digits


disp "Dickey-Fuller tau statistic" %tstats(2)


Dickey-Fuller tau statistic      -1.44870

* The DFUNIT procedure can be employed to do the test as well. The
* option LAGS indicates the number of additional lags of the differenced
* dependent variable to include. Note that the critical values are
* slightly different from those shown in the text. The text is providing
* asymptotical critical values, while the procedure show critical values
* corrected for sample size.


@dfunit(lags=2) x


Dickey-Fuller Unit Root Test, Series X
Regression Run From 4 to 200
Observations        198
With intercept
Using 2 lags on the differences
```

```
Sig Level    Crit Value
1%(**)         -3.46454
5%(*)          -2.87614
10%            -2.57448

T-Statistic    -1.44870

* To do the test without an intercept, use the option DET=NONE. (DET is
* short for DETERMINISTIC). You can also use DET=TREND for the variation
* of the test which includes 1 and time as regressors.

@dfunit(lags=2,det=none) x


Dickey-Fuller Unit Root Test, Series X
Regression Run From 4 to 200
Observations        198
Without intercept or trend
Using 2 lags on the differences

Sig Level    Crit Value
1%(**)         -2.57596
5%(*)          -1.94131
10%            -1.61651

T-Statistic    -0.65849
*
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 6.3.2 on pp 197-198
```

```
open data oshorts.dat
data(format=free,org=columns) 1 57 overs

boxjenk(ma=1,maxl,demean) overs
compute unrlogl=%logl
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     9 Iterations. Final criterion was  0.0000073 <=  0.0000100
Dependent Variable OVERS
Usable Observations                      57
Degrees of Freedom                       56
Centered R^2                      0.4025114
R-Bar^2                           0.4025114
Uncentered R^2                    0.4053460
Mean of Dependent Variable      -4.03508772
Std Error of Dependent Variable  58.96365858
Standard Error of Estimate       45.57736783
Sum of Squared Residuals       116328.60166
Log Likelihood                    -298.6336
Durbin-Watson Statistic              1.9554
Q(14-1)                             13.4695
Significance Level of Q           0.4122380

    Variable                       Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  MA{1}                       -0.817631615  0.080494607    -10.15760  0.00000000
```

```
disp "Davis-Dunsmuir test" %beta(1) "vs .05 critical value" -1.0+6.80/%nobs
```

```
Davis-Dunsmuir test      -0.81763 vs .05 critical value      -0.88070
* To do the likelihood ratio version of the test, we need to evaluate the
* likelihood at the hypothesized value (theta=-1). This is done by using
* METHOD=EVAL (which evaluates, rather than estimating) and feeding in
* the value with the INITIAL option.

disp "Davis-Dunsmuir test" %beta(1) "vs .05 critical value" -1.0+6.80/%nobs
Davis-Dunsmuir test      -0.81763 vs .05 critical value      -0.88070
boxjenk(ma=1,maxl,demean,initial=||-1.0||,method=eval) overs
compute rlogl=%logl
disp "LR version of test" -2.0*(rlogl-unrlogl)

LR version of test       7.31705
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 6.4.1 from page 202
open data dowj.dat
data(format=free,org=columns) 1 78 dowj
* In order to forecast the original (undifferenced) series easily, the
* DIFFS option is used on the BOXJENK instruction. We also use the
* DEFINE option to define a forecasting equation.
boxjenk(diffs=1,ar=1,demean,maxl,define=doweq) dowj


Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     1 Iterations. Final criterion was  0.0000000 <=  0.0000100
Dependent Variable DOWJ
Usable Observations                    77
Degrees of Freedom                     76
Centered R^2                    0.9940068
```

```
R-Bar^2                              0.9940068
Uncentered R^2                       0.9999866
Mean of Dependent Variable         115.74493506
Std Error of Dependent Variable      5.51504655
Standard Error of Estimate           0.42695035
Sum of Squared Residuals            13.853781818
Log Likelihood                        -43.2213
Durbin-Watson Statistic                 1.0867
Q(19-1)                                46.2524
Significance Level of Q              0.0002725

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                        0.8000000000 0.0000000000      0.00000  0.00000000

* The numbering scheme for the forecasts in the text is going to be
* slightly different from the one used here. With 78 data points, the
* first two out-of-sample periods will be 79 and 80.

uforecast(equation=doweq,stderrs=ferr) forecast 79 80

* This prints the forecasts and the standard errors (square root of the
* variance).
*
print / forecast ferr

 ENTRY      FORECAST          FERR
```

```
     79   120.6407272727  0.426950351763

     80   120.1960363636  0.879144582064
```

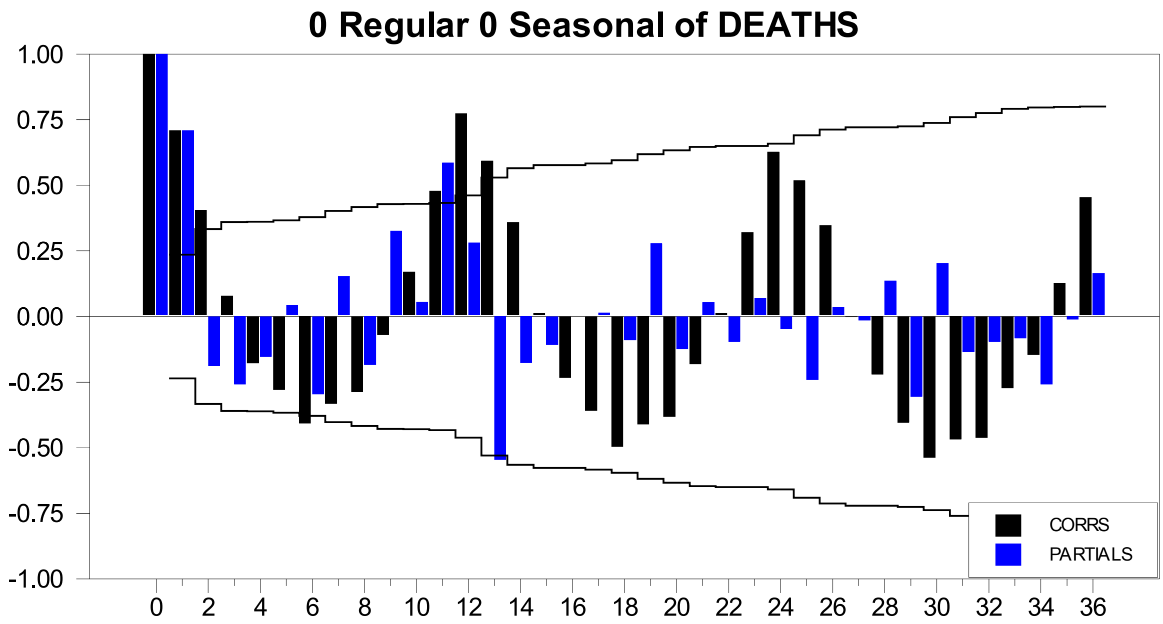
```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 6.5.4 from pp 206-208
open data deaths.dat
calendar(m) 1973
data(format=free,org=columns) 1973:1 1978:12 deaths
print
```

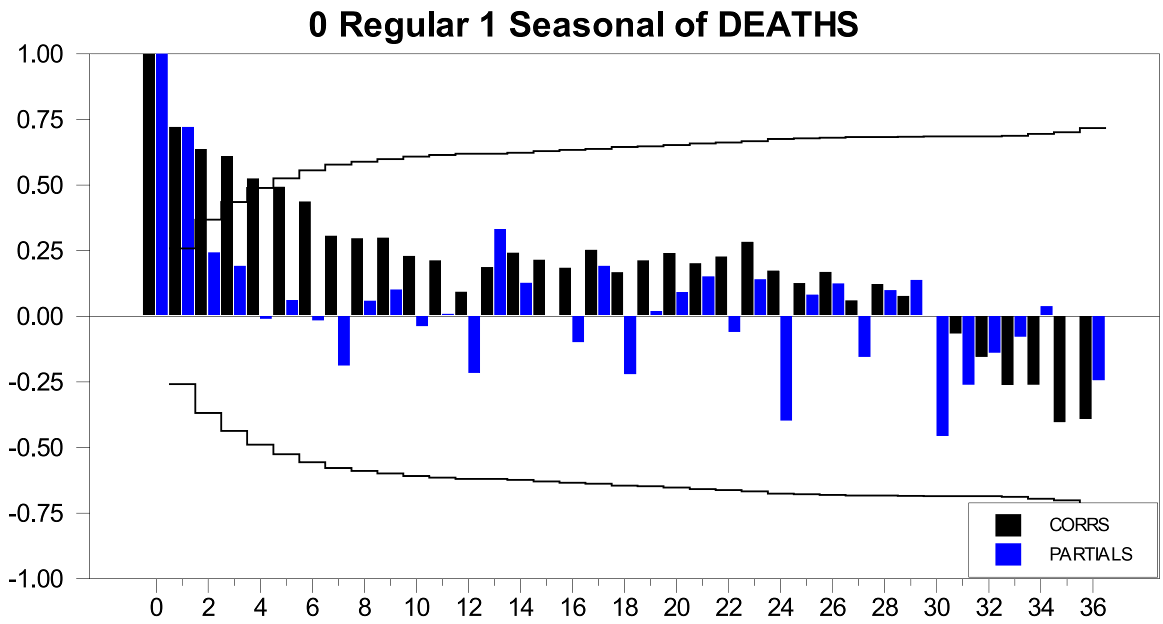
ENTRY	DEATHS
1973:01	9007
1973:02	8106
1973:03	8928
1973:04	9137
1973:05	10017
1973:06	10826
1973:07	11317
1973:08	10744
1973:09	9713
1973:10	9938
1973:11	9161
1973:12	8927
1974:01	7750
1974:02	6981
1974:03	8038
1974:04	8422

1974:05	8714
1974:06	9512
1974:07	10120
1974:08	9823
1974:09	8743
1974:10	9129
1974:11	8710
1974:12	8680
1975:01	8162
1975:02	7306
1975:03	8124
1975:04	7870
1975:05	9387
1975:06	9556
1975:07	10093
1975:08	9620
1975:09	8285
1975:10	8433
1975:11	8160
1975:12	8034
1976:01	7717
1976:02	7461
1976:03	7776
1976:04	7925
1976:05	8634
1976:06	8945
1976:07	10078
1976:08	9179
1976:09	8037

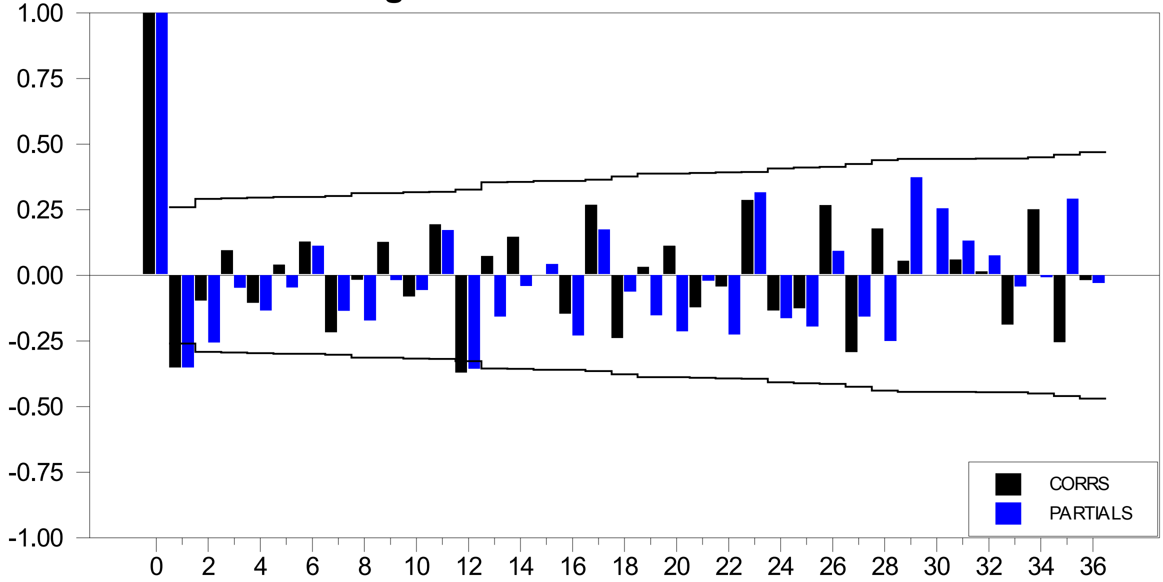
1976:10	8488
1976:11	7874
1976:12	8647
1977:01	7792
1977:02	6957
1977:03	7726
1977:04	8106
1977:05	8890
1977:06	9299
1977:07	10625
1977:08	9302
1977:09	8314
1977:10	8850
1977:11	8265
1977:12	8796
1978:01	7836
1978:02	6892
1978:03	7791
1978:04	8129
1978:05	9115
1978:06	9434
1978:07	10484
1978:08	9827
1978:09	9110
1978:10	9070
1978:11	8633
1978:12	9240

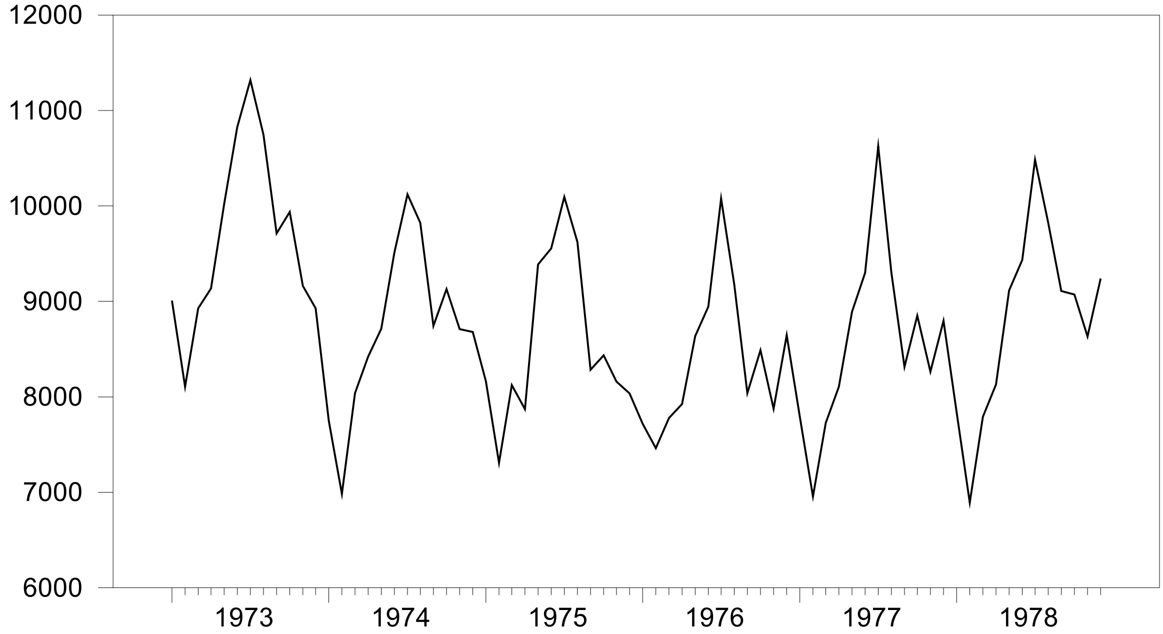
```
Use BJIDENT to get the autocorrelations and partial autocorrelations
* of the various combinations.
```





1 Regular 1 Seasonal of DEATHS





```
* Estimate the model with first and seasonal differencing. To get the multiplicative
structure with RATS, you just need to indicate one MAand one SMA (seasonal MA).

boxjenk(diff=1,sdiff=1,demean,ma=1,maxl) deaths

@bdindtests %resids

boxjenk(diff=1,sdiff=1,demean,ma=1,maxl) deaths

@bdindtests %resids

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     9 Iterations. Final criterion was  0.0000056 <=  0.0000100

Dependent Variable DEATHS

Monthly Data From 1974:02 To 1978:12

Usable Observations                        59

Degrees of Freedom                         58

Centered R^2                        0.8336460

R-Bar^2                             0.8336460

Uncentered R^2                      0.9983590

Mean of Dependent Variable       8629.5932203

Std Error of Dependent Variable   868.7347343

Standard Error of Estimate        354.3265902

Sum of Squared Residuals         7281745.2876

Log Likelihood                      -429.7164

Durbin-Watson Statistic                1.9251

Q(14-1)                               13.1639

Significance Level of Q             0.4352339

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  MA{1}                        -0.523865231  0.112548294     -4.65458   0.00001932
```

```
Independence Tests for Series %RESIDS

Test            Statistic  P-Value

Ljung-Box Q(14)  16.695322     0.2728

McLeod-Li(14)     8.419257     0.8664

Turning Points   -0.627250     0.5305

Difference Sign   0.447214     0.6547

Rank Test        -0.738962     0.4599
```

Estimate the full 13 lag MA model

```
boxjenk(diff=1,sdiff=1,demean,ma=13,iters=80) deaths
```

```
Box-Jenkins - Estimation by LS Gauss-Newton

Convergence in    24 Iterations. Final criterion was  0.0000066 <=  0.0000100

Dependent Variable DEATHS

Monthly Data From 1974:02 To 1978:12

Usable Observations                        59

Degrees of Freedom                         46

Centered R^2                        0.9014156

R-Bar^2                             0.8756980

Uncentered R^2                      0.9990275

Mean of Dependent Variable       8629.5932203

Std Error of Dependent Variable   868.7347343

Standard Error of Estimate        306.2854215

Sum of Squared Residuals         4315294.9322

Log Likelihood                      -414.1215

Durbin-Watson Statistic                1.9736

Q(14-13)                               4.7995

Significance Level of Q             0.0284675
```

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	-0.473059224	0.137384739	-3.44332	0.00123514
2. MA{2}	0.147609545	0.121502952	1.21486	0.23061733
3. MA{3}	-0.216105973	0.123275772	-1.75303	0.08625954
4. MA{4}	-0.104366537	0.129315417	-0.80707	0.42377990
5. MA{5}	0.167994018	0.131002225	1.28238	0.20613594
6. MA{6}	-0.180135685	0.135009773	-1.33424	0.18869243
7. MA{7}	-0.242800419	0.139557376	-1.73979	0.08858368
8. MA{8}	-0.096714053	0.143765754	-0.67272	0.50449118
9. MA{9}	0.151183481	0.145676310	1.03780	0.30478594
10. MA{10}	0.208525963	0.148403640	1.40513	0.16670121
11. MA{11}	0.065613277	0.147805175	0.44392	0.65918364
12. MA{12}	-0.803680147	0.150040672	-5.35642	0.00000263
13. MA{13}	0.438869422	0.163955683	2.67676	0.01026788

Estimate a subset model. This runs into some problems with roots on the unit circle in the MA polynomial

```
boxjenk(diff=1,sdiff=1,demean,ma=||1,6,12,13||,maxl) deaths
```

Box-Jenkins - Estimation by ML Gauss-Newton

NO CONVERGENCE IN 8 ITERATIONS

LAST CRITERION WAS 0.0000000

SUBITERATIONS LIMIT EXCEEDED.

ESTIMATION POSSIBLY HAS STALLED OR MACHINE ROUNDOFF IS MAKING FURTHER PROGRESS DIFFICULT

TRY HIGHER SUBITERATIONS LIMIT, TIGHTER CVCRIT, DIFFERENT SETTING FOR EXACTLINE OR ALPHA ON NLPAR

```
RESTARTING ESTIMATION FROM LAST ESTIMATES OR DIFFERENT INITIAL GUESSES MIGHT ALSO WORK
Dependent Variable DEATHS
Monthly Data From 1974:02 To 1978:12
Usable Observations                        59
Degrees of Freedom                         55
Centered R^2                        0.8818561
R-Bar^2                             0.8754118
Uncentered R^2                      0.9988346
Mean of Dependent Variable       8629.5932203
Std Error of Dependent Variable   868.7347343
Standard Error of Estimate        306.6377057
Sum of Squared Residuals         5171467.5410
Log Likelihood                      -424.0611
Durbin-Watson Statistic                2.1567
Q(14-4)                               11.3536
Significance Level of Q             0.3306393

    Variable                        Coeff      Std Error      T-Stat      Signif
**********************************************************************************
1.  MA{1}                       -0.389989647  0.138835266     -2.80901  0.00686688
2.  MA{6}                       -0.089144531  0.208986989     -0.42656  0.67136818
3.  MA{12}                      -0.674545106  0.210969003     -3.19737  0.00230052
4.  MA{13}                       0.077404566  0.147275657      0.52558  0.60129542


* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 6.5.5 from page 210
open data deaths.dat
calendar(m) 1973
```

```
data(format=free,org=columns) 1973:1 1978:12 deaths

boxjenk(diff=1,sdiff=1,demean,ma=1,sma=1,maxl,define=eq658) deaths

uforecast(equation=eq658,stderrs=s658) f658 1979:1 1979:6
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in    12 Iterations. Final criterion was  0.0000049 <=  0.0000100
Dependent Variable DEATHS
Monthly Data From 1974:02 To 1978:12
Usable Observations                      59
Degrees of Freedom                       57
Centered R^2                      0.8722632
R-Bar^2                           0.8700222
Uncentered R^2                    0.9987400
Mean of Dependent Variable     8629.5932203
Std Error of Dependent Variable  868.7347343
Standard Error of Estimate       313.2000183
Sum of Squared Residuals       5591372.3326
Log Likelihood                    -424.4658
Durbin-Watson Statistic              1.9296
Q(14-2)                             12.1904
Significance Level of Q           0.4305104

    Variable                      Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  MA{1}                    -0.483088768  0.113435896     -4.25869  0.00007775
2.  SMA{12}                  -0.591339713  0.150621275     -3.92600  0.00023532
```

```
boxjenk(diff=1,sdiff=1,demean,ma=||1,6,12,13||,maxl,define=eq659) deaths
```

```
uforecast(equation=eq659,stderrs=s659) f659 1979:1 1979:6
```

```
Box-Jenkins - Estimation by ML Gauss-Newton

NO CONVERGENCE IN 8 ITERATIONS

LAST CRITERION WAS  0.0000000

SUBITERATIONS LIMIT EXCEEDED.

ESTIMATION POSSIBLY HAS STALLED OR MACHINE ROUNDOFF IS MAKING FURTHER PROGRESS
DIFFICULT

TRY HIGHER SUBITERATIONS LIMIT, TIGHTER CVCRIT, DIFFERENT SETTING FOR EXACTLINE OR
ALPHA ON NLPAR

RESTARTING ESTIMATION FROM LAST ESTIMATES OR DIFFERENT INITIAL GUESSES MIGHT ALSO WORK

Dependent Variable DEATHS

Monthly Data From 1974:02 To 1978:12

Usable Observations                     59

Degrees of Freedom                      55

Centered R^2                     0.8818561

R-Bar^2                          0.8754118

Uncentered R^2                   0.9988346

Mean of Dependent Variable    8629.5932203

Std Error of Dependent Variable  868.7347343

Standard Error of Estimate     306.6377057

Sum of Squared Residuals      5171467.5410

Log Likelihood                   -424.0611

Durbin-Watson Statistic             2.1567

Q(14-4)                            11.3536

Significance Level of Q          0.3306393

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
```

```
1.  MA{1}                        -0.389989647  0.138835266     -2.80901  0.00686688
2.  MA{6}                        -0.089144531  0.208986989     -0.42656  0.67136818
3.  MA{12}                       -0.674545106  0.210969003     -3.19737  0.00230052
4.  MA{13}                        0.077404566  0.147275657      0.52558  0.60129542
```

```
print(picture="*.") / f658 s658 f659 s659
```

ENTRY	F658	S658	F659	S659
1979:01	8442	313	8364	307
1979:02	7712	353	7649	359
1979:03	8558	388	8504	405
1979:04	8887	420	8866	446
1979:05	9851	450	9838	484
1979:06	10295	479	10277	519

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 6.6.2 from pp 215-217
open data lake.dat
cal 1875
data(format=free,org=columns) 1875:1 1972:1 lake
Estimate the time trend model and get the residuals
set trend = t
linreg lake / w
# constant trend
```

Linear Regression - Estimation by Least Squares

```
Dependent Variable LAKE
Annual Data From 1875:01 To 1972:01
Usable Observations                     98
Degrees of Freedom                      96
Centered R^2                     0.2724728
R-Bar^2                          0.2648943
Uncentered R^2                   0.9848844
Mean of Dependent Variable    9.0040816327
Std Error of Dependent Variable  1.3182985260
Standard Error of Estimate    1.1302867788
Sum of Squared Residuals      122.64462743
Regression F(1,96)                 35.9538
Significance Level of F          0.0000000
Log Likelihood                   -150.0478
Durbin-Watson Statistic             0.4395

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  Constant                     10.20203661   0.23011125     44.33524  0.00000000
2.  TREND                        -0.02420111   0.00403611     -5.99615  0.00000004
```

See how an AR(2) model does fitting those

```
boxjenk(ar=2,maxl) w
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     5 Iterations. Final criterion was  0.0000008 <=  0.0000100
Dependent Variable W
Annual Data From 1875:01 To 1972:01
Usable Observations                     98
```

```
Degrees of Freedom                        96
Centered R^2                       0.6347102
R-Bar^2                            0.6309051
Uncentered R^2                     0.6347102
Mean of Dependent Variable      0.0000000000
Std Error of Dependent Variable 1.1244454644
Standard Error of Estimate      0.6831363442
Sum of Squared Residuals        44.800825420
Log Likelihood                     -101.2551
Durbin-Watson Statistic               1.9596
Q(24-2)                              10.2089
Significance Level of Q            0.9843099

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                        1.005012680  0.097396576     10.31877  0.00000000
2.  AR{2}                       -0.292474597  0.099093489     -2.95150  0.00397433
```

Estimate the combined model

```
boxjenk(ar=2,reg,maxl) lake
# constant trend
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     6 Iterations. Final criterion was  0.0000067 <=  0.0000100
Dependent Variable LAKE
Annual Data From 1875:01 To 1972:01
```

```
Usable Observations                            98
Degrees of Freedom                             94
Centered R^2                            0.7345516
R-Bar^2                                 0.7260798
Uncentered R^2                          0.9944849
Mean of Dependent Variable           9.0040816327
Std Error of Dependent Variable 1.3182985260
Standard Error of Estimate           0.6899629842
Sum of Squared Residuals             44.748598436
Log Likelihood                          -101.1983
Durbin-Watson Statistic                    1.9581
Q(24-2)                                   10.4832
Significance Level of Q                 0.9813689

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                          1.00481732   0.09839391     10.21219  0.00000000
2.  AR{2}                         -0.29130092   0.10029857     -2.90434  0.00458723
3.  Constant                      10.09154709   0.47172393     21.39291 0.00000000
4.  TREND                         -0.02156816   0.00825347     -2.61322  0.01044422
```

Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

Example 7.1.1 from pp 225-228

```
open data djao2.dat
data(format=prn,org=columns) 1 251 dj ao
print
```

ENTRY	DJ	AO
1	3621.63	1941.5
2	3634.21	1938.3
3	3615.76	1912.9
4	3633.65	1903.6
5	3630.85	1902.6
6	3613.25	1925.5
7	3575.80	1924.1
8	3537.24	1925.2
9	3547.02	1919.3
10	3539.75	1928.6
11	3543.11	1946.5
12	3567.70	1943.0
13	3566.02	1942.3
14	3566.30	1951.4
15	3555.12	1964.4
16	3581.11	1972.7
17	3577.76	1977.0
18	3587.26	1998.5
19	3598.99	2018.8
20	3583.63	2022.5
21	3584.74	2026.2
22	3593.41	2039.8
23	3593.13	2028.0
24	3603.19	2038.6
25	3621.63	2062.0
26	3629.73	2074.1
27	3642.31	2085.5
28	3635.32	2075.5

29	3645.10	2051.7
30	3636.16	2060.4
31	3649.30	2061.4
32	3673.61	2046.9
33	3672.49	2055.7
34	3664.66	2068.3
35	3687.86	2076.3
36	3680.59	2112.2
37	3692.61	2132.4
38	3697.64	2125.3
39	3661.87	2108.4
40	3624.98	2101.6
41	3643.43	2079.9
42	3647.90	2054.2
43	3640.07	2050.8
44	3663.55	2042.9
45	3662.43	2052.4
46	3684.51	2074.0
47	3677.52	2082.9
48	3710.77	2083.8
49	3704.35	2104.3
50	3685.34	2108.0
51	3694.01	2083.2
52	3670.25	2049.3
53	3674.17	2009.6
54	3687.58	2032.4
55	3687.58	2042.0
56	3683.95	2043.1
57	3677.80	2010.3

58	3683.95	2009.4
59	3697.08	2005.4
60	3702.11	2047.3
61	3704.07	2047.4
62	3710.21	2053.7
63	3718.88	2073.9
64	3734.53	2096.0
65	3729.78	2095.7
66	3740.67	2084.9
67	3764.43	2094.5
68	3742.63	2086.6
69	3716.92	2069.9
70	3726.14	2074.8
71	3751.57	2080.2
72	3755.21	2076.0
73	3745.15	2067.0
74	3762.19	2053.2
75	3757.72	2068.8
76	3757.72	2089.2
77	3792.93	2089.2
78	3793.77	2089.2
79	3794.33	2126.9
80	3775.88	2154.5
81	3754.09	2173.6
82	3756.60	2173.6
83	3783.90	2174.3
84	3798.82	2193.4
85	3803.88	2200.3
86	3820.77	2186.0

87	3865.51	2198.6
88	3850.31	2206.7
89	3848.63	2195.6
90	3842.43	2177.5
91	3867.20	2206.4
92	3870.29	2238.2
93	3870.29	2232.1
94	3884.37	2248.2
95	3891.96	2266.2
96	3914.48	2250.3
97	3912.79	2224.5
98	3895.34	2221.9
99	3908.00	2221.9
100	3926.30	2250.7
101	3945.43	2259.9
102	3978.36	2310.8
103	3964.01	2310.1
104	3975.54	2312.1
105	3967.66	2340.6
106	3871.42	2332.8
107	3906.32	2281.1
108	3906.03	2305.4
109	3931.92	2270.9
110	3895.34	2234.3
111	3894.78	2241.4
112	3904.06	2238.6
113	3928.27	2234.0
114	3937.27	2249.0
115	3922.64	2240.9

116	3887.46	2223.2
117	3887.46	2178.5
118	3911.66	2202.5
119	3891.68	2218.9
120	3839.90	2197.0
121	3838.78	2148.8
122	3832.02	2180.1
123	3809.23	2181.7
124	3831.74	2154.0
125	3824.42	2151.4
126	3832.30	2116.8
127	3856.22	2144.7
128	3851.72	2171.7
129	3853.41	2146.8
130	3830.62	2155.1
131	3862.70	2153.1
132	3862.98	2179.3
133	3849.59	2172.5
134	3848.15	2173.5
135	3865.14	2164.4
136	3895.65	2163.5
137	3864.85	2140.5
138	3862.55	2140.8
139	3869.46	2180.9
140	3821.09	2169.8
141	3774.73	2151.6
142	3762.35	2108.9
143	3699.02	2100.8
144	3626.75	2092.4

145	3635.96	2053.1
146	3635.96	2053.1
147	3593.35	2053.1
148	3675.41	2050.0
149	3679.73	2084.1
150	3693.26	2087.4
151	3674.26	2082.0
152	3688.83	2076.0
153	3681.69	2095.1
154	3661.47	2111.2
155	3663.25	2095.0
156	3661.47	2080.6
157	3620.42	2095.9
158	3619.82	2061.4
159	3598.71	2046.6
160	3652.54	2029.6
161	3648.68	2042.5
162	3705.78	2042.5
163	3699.54	2069.4
164	3699.54	2059.7
165	3668.31	2069.1
166	3681.69	2066.1
167	3701.02	2047.9
168	3714.41	2044.2
169	3697.75	2018.4
170	3695.97	1988.1
171	3669.50	2004.3
172	3629.04	2009.3
173	3656.41	2008.2

174	3629.04	2034.6
175	3652.84	2041.4
176	3659.68	2070.0
177	3671.50	2110.9
178	3720.61	2096.0
179	3732.89	2107.8
180	3758.98	2093.7
181	3766.35	2103.9
182	3742.41	2121.0
183	3745.17	2132.4
184	3755.30	2105.9
185	3753.46	2096.9
186	3757.14	2102.2
187	3757.14	2091.8
188	3758.37	2081.8
189	3760.83	2097.2
190	3758.99	2077.0
191	3772.22	2078.6
192	3768.52	2072.5
193	3755.91	2070.2
194	3749.45	2079.7
195	3753.14	2076.7
196	3773.45	2069.4
197	3783.12	2069.4
198	3814.83	2076.6
199	3790.41	2074.4
200	3811.34	2056.0
201	3776.78	2051.2
202	3741.90	2024.4

203	3707.97	1993.6
204	3724.77	2010.9
205	3699.09	2022.5
206	3636.94	2017.9
207	3685.50	1957.4
208	3669.64	1974.4
209	3667.05	1975.1
210	3624.96	1989.1
211	3646.65	1965.8
212	3646.65	1987.1
213	3652.48	2003.4
214	3674.50	1991.2
215	3688.42	1962.2
216	3709.14	1964.9
217	3702.99	1961.2
218	3702.66	1972.9
219	3704.28	1978.6
220	3739.25	2007.7
221	3753.81	2058.0
222	3755.43	2072.3
223	3748.31	2077.4
224	3727.27	2078.6
225	3732.45	2049.2
226	3735.04	2052.5
227	3741.84	2048.3
228	3735.68	2041.3
229	3720.47	2041.7
230	3730.83	2042.1
231	3764.50	2061.5

```
    232     3798.17          2082.1
    233     3796.22          2086.9
    234     3792.16          2072.3
    235     3765.79          2083.5
    236     3747.02          2091.9
    237     3753.81          2081.1
    238     3755.76          2086.8
    239     3766.76          2076.5
    240     3750.90          2062.8
    241     3768.71          2051.9
    242     3760.29          2055.7
    243     3784.57          2040.0
    244     3776.48          2059.5
    245     3755.43          2066.8
    246     3755.11          2061.3
    247     3751.22          2063.6
    248     3775.83          2051.6
    249     3846.73          2061.1
    250     3829.89          2077.8
    251     3881.05          2077.2


graph(footer="Figure 7.1 Dow Jones Index and Australian All Ordinaries Index") 2
# dj
# ao
```

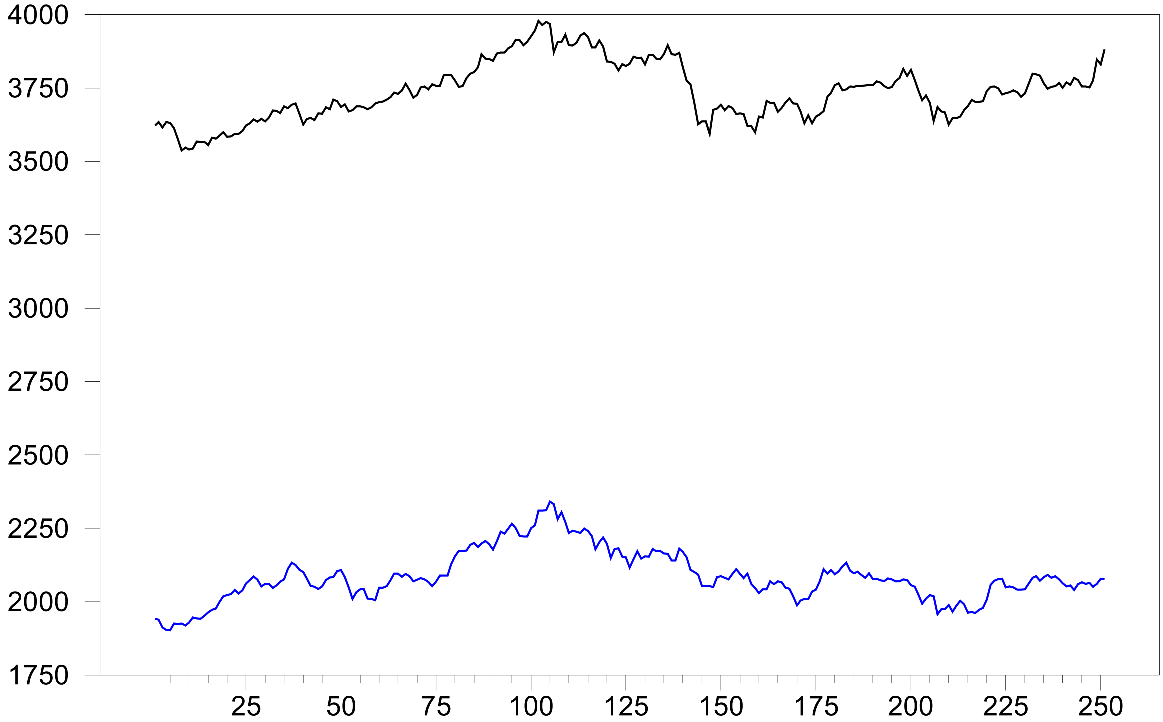
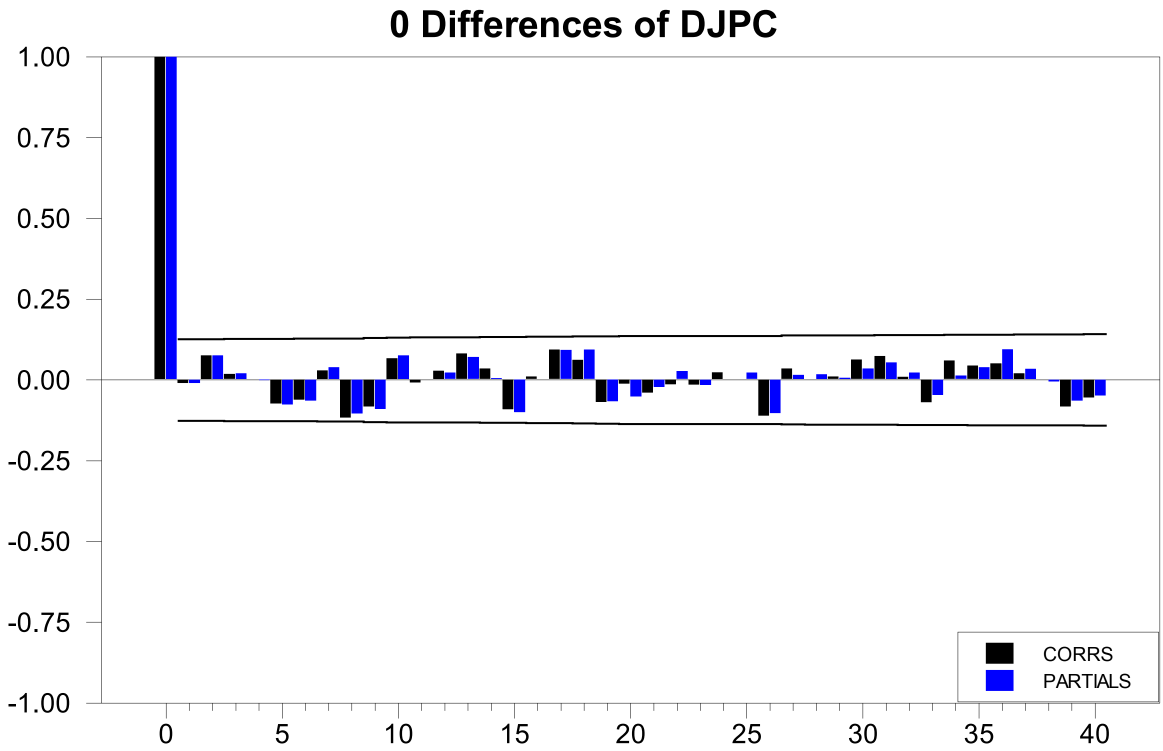
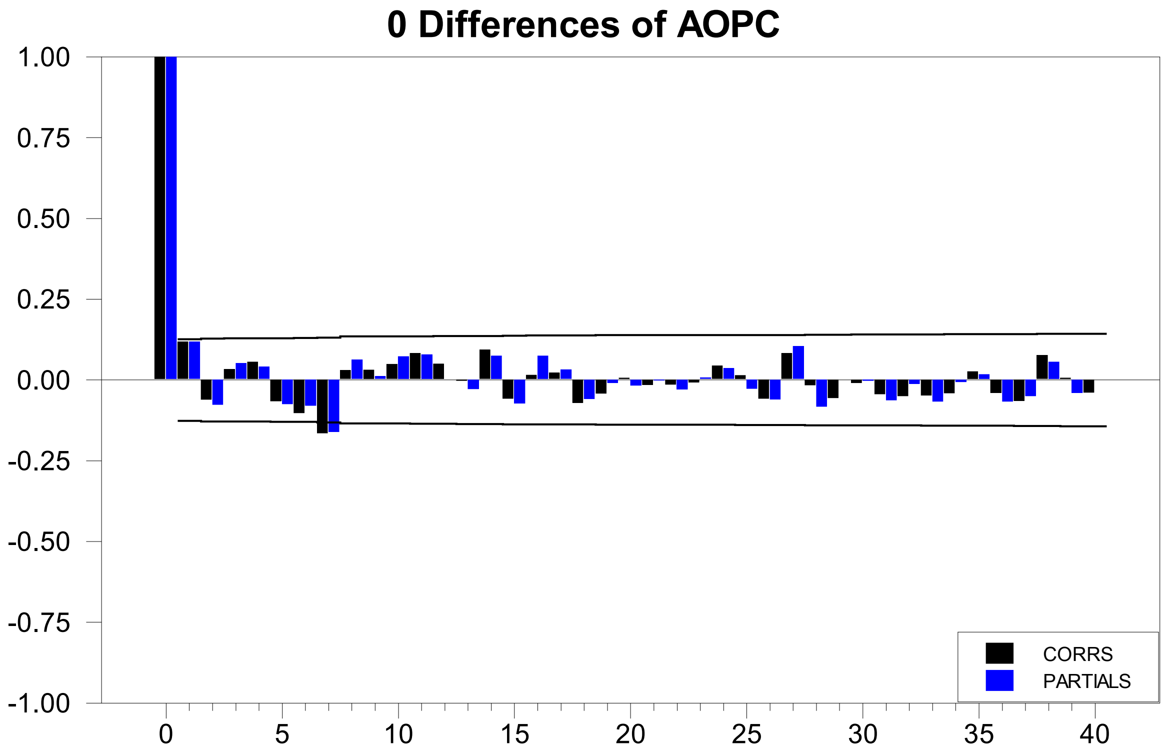


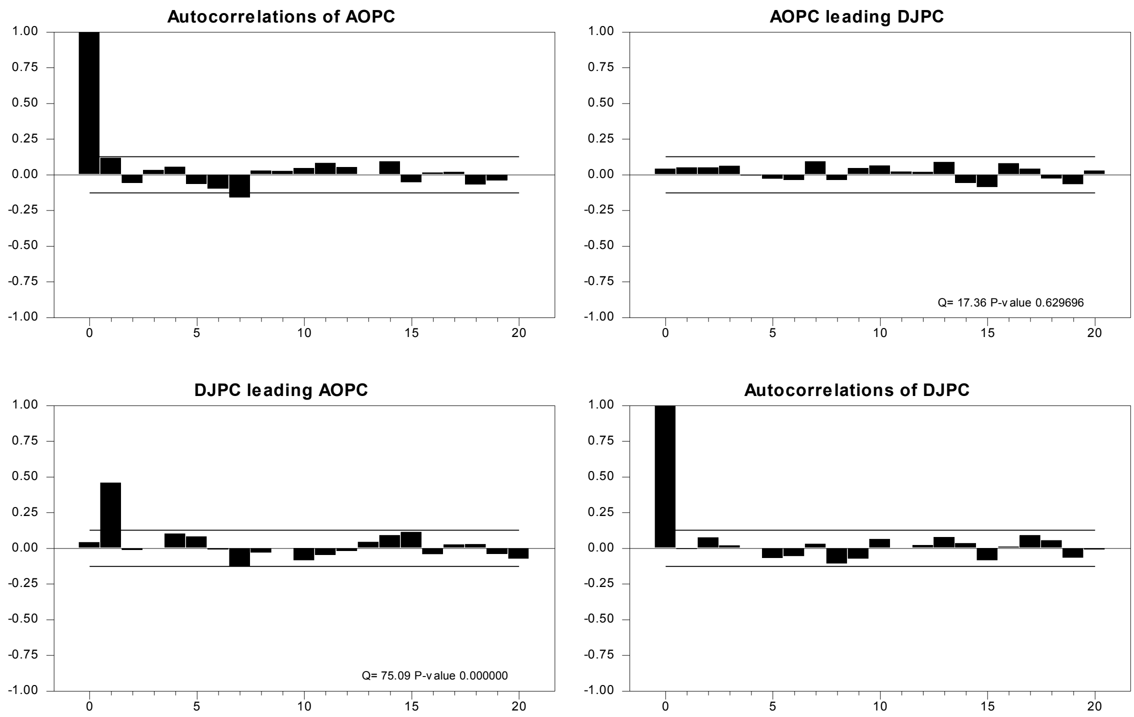
Figure 7.1 Dow Jones Index and Australian All Ordinaries Index




```
set djpc = 100*(dj-dj{1})/dj{1}

set aopc = 100*(ao-ao{1})/ao{1}

@bjident(diffs=0,number=40) djpc

@bjident(diffs=0,number=40) aopc
```




```
@crosscorr(number=20) aopc djpc
```




```
set djpclag = djpc{1}

scatter(footer="Figure 7.5 Scatterplot of djpc lagged vs aopc")

# djpclag aopc
```

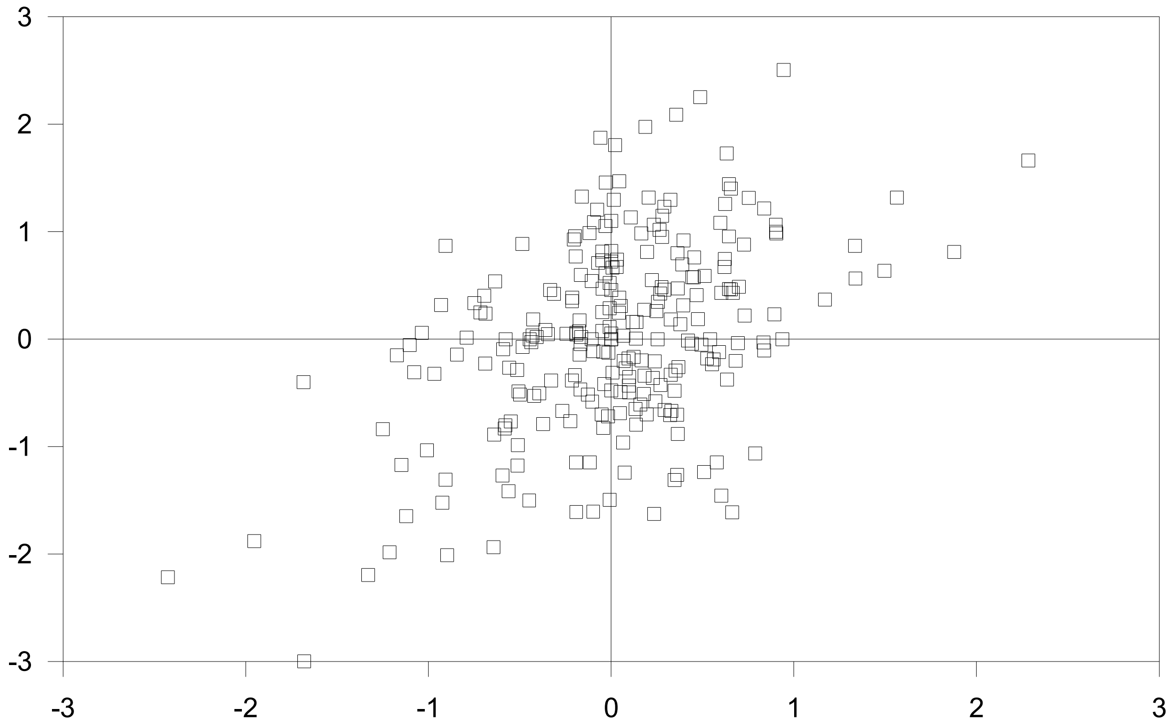


Figure 7.5 Scatterplot of djpc lagged vs aopc

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 7.1.2 from pp 228-229

open data ls2.tsm

data(format=free,org=columns) 1 150 lead sales

print
```

ENTRY	LEAD	SALES
1	10.01	200.1
2	10.07	199.5
3	10.32	199.4
4	9.75	198.9
5	10.33	199.0

6	10.13	200.2
7	10.36	198.6
8	10.32	200.0
9	10.13	200.3
10	10.16	201.2
11	10.58	201.6
12	10.62	201.5
13	10.86	201.5
14	11.20	203.5
15	10.74	204.9
16	10.56	207.1
17	10.48	210.5
18	10.77	210.5
19	11.33	209.8
20	10.96	208.8
21	11.16	209.5
22	11.70	213.2
23	11.39	213.7
24	11.42	215.1
25	11.94	218.7
26	11.24	219.8
27	11.59	220.5
28	10.96	223.8
29	11.40	222.8
30	11.02	223.8
31	11.01	221.7
32	11.23	222.3
33	11.33	220.8
34	10.83	219.4

35	10.84	220.1
36	11.14	220.6
37	10.38	218.9
38	10.90	217.8
39	11.05	217.7
40	11.11	215.0
41	11.01	215.3
42	11.22	215.9
43	11.21	216.7
44	11.91	216.7
45	11.69	217.7
46	10.93	218.7
47	10.99	222.9
48	11.01	224.9
49	10.84	222.2
50	10.76	220.7
51	10.77	220.0
52	10.88	218.7
53	10.49	217.0
54	10.50	215.9
55	11.00	215.8
56	10.98	214.1
57	10.61	212.3
58	10.48	213.9
59	10.53	214.6
60	11.07	213.6
61	10.61	212.1
62	10.86	211.4
63	10.34	213.1

64	10.78	212.9
65	10.80	213.3
66	10.33	211.5
67	10.44	212.3
68	10.50	213.0
69	10.75	211.0
70	10.40	210.7
71	10.40	210.1
72	10.34	211.4
73	10.55	210.0
74	10.46	209.7
75	10.82	208.8
76	10.91	208.8
77	10.87	208.8
78	10.67	210.6
79	11.11	211.9
80	10.88	212.8
81	11.28	212.5
82	11.27	214.8
83	11.44	215.3
84	11.52	217.5
85	12.10	218.8
86	11.83	220.7
87	12.62	222.2
88	12.41	226.7
89	12.43	228.4
90	12.73	233.2
91	13.01	235.7
92	12.74	237.1

93	12.73	240.6
94	12.76	243.8
95	12.92	245.3
96	12.64	246.0
97	12.79	246.3
98	13.05	247.7
99	12.69	247.6
100	13.01	247.8
101	12.90	249.4
102	13.12	249.0
103	12.47	249.9
104	12.47	250.5
105	12.94	251.5
106	13.10	249.0
107	12.91	247.6
108	13.39	248.8
109	13.13	250.4
110	13.34	250.7
111	13.34	253.0
112	13.14	253.7
113	13.49	255.0
114	13.87	256.2
115	13.39	256.0
116	13.59	257.4
117	13.27	260.4
118	13.70	260.0
119	13.20	261.3
120	13.32	260.4
121	13.15	261.6

122	13.30	260.8
123	12.94	259.8
124	13.29	259.0
125	13.26	258.9
126	13.08	257.4
127	13.24	257.7
128	13.31	257.9
129	13.52	257.4
130	13.02	257.3
131	13.25	257.6
132	13.12	258.9
133	13.26	257.8
134	13.11	257.7
135	13.30	257.2
136	13.06	257.5
137	13.32	256.8
138	13.10	257.5
139	13.27	257.0
140	13.64	257.6
141	13.58	257.3
142	13.87	257.5
143	13.53	259.6
144	13.41	261.1
145	13.25	262.9
146	13.50	263.3
147	13.58	262.8
148	13.51	261.8
149	13.77	262.2
150	13.40	262.7

First difference and demean series

```
set dsales = sales-sales{1}
set dlead  = lead-lead{1}
diff(center) dsales / csales
compute salesmean=%mean
diff(center) dlead  / clead
compute leadmean=%mean
```

Fit the MA(1) model to the lead variable

```
boxjenk(ma=1,maxl) clead
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in    10 Iterations. Final criterion was  0.0000075 <=  0.0000100
Dependent Variable CLEAD
Usable Observations                        149
Degrees of Freedom                         148
Centered R^2                         0.2153471
R-Bar^2                              0.2153471
Uncentered R^2                       0.2153471
Mean of Dependent Variable        0.0000000000
Std Error of Dependent Variable   0.3162253282
Standard Error of Estimate        0.2801144044
Sum of Squared Residuals          11.612683771
```

```
Log Likelihood                      -21.4366
Durbin-Watson Statistic               2.0999
Q(36-1)                              46.7195
Significance Level of Q            0.0889844

    Variable                       Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  MA{1}                       -0.474354402  0.072581465     -6.53548  0.00000000
```

And an ARMA(1,1) model to the sales variable

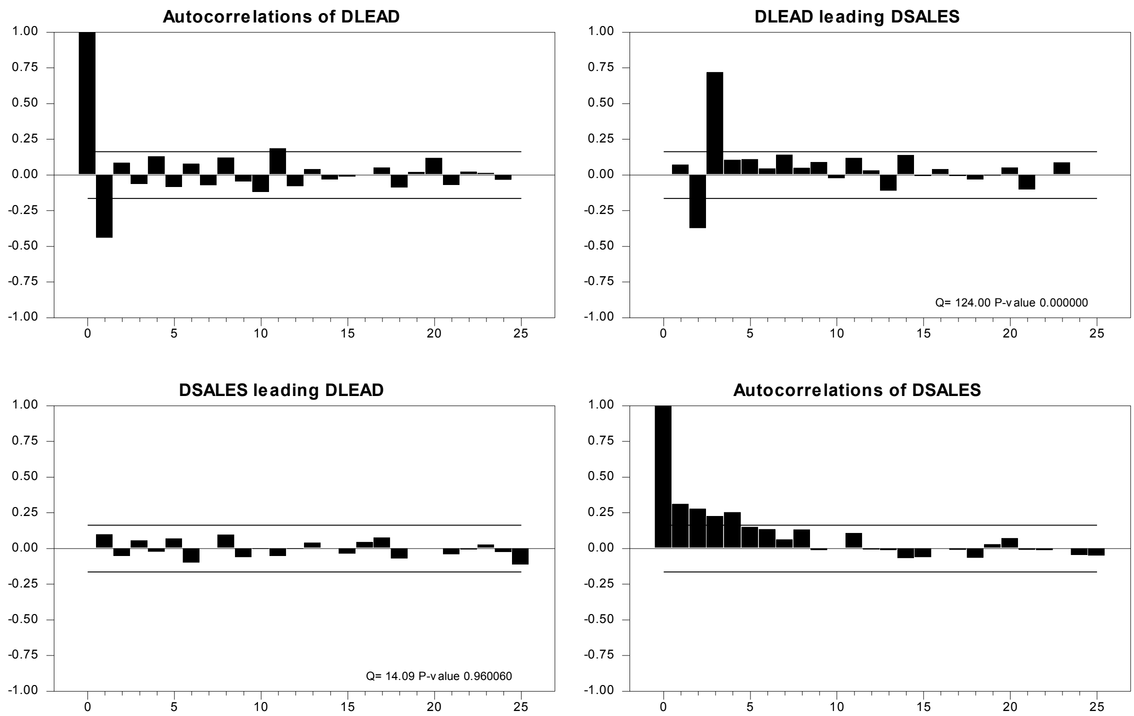
```
boxjenk(ma=1,ar=1,maxl) csales
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in    11 Iterations. Final criterion was 0.0000080 <= 0.0000100
Dependent Variable CSALES
Usable Observations                      149
Degrees of Freedom                       147
Centered R^2                       0.1532523
R-Bar^2                            0.1474921
Uncentered R^2                     0.1532523
Mean of Dependent Variable      0.0000000000
Std Error of Dependent Variable 1.4439987584
Standard Error of Estimate      1.3332636044
Sum of Squared Residuals        261.30600030
Log Likelihood                     -253.3949
Durbin-Watson Statistic               2.0312
Q(36-2)                              35.5891
Significance Level of Q            0.3934146
```

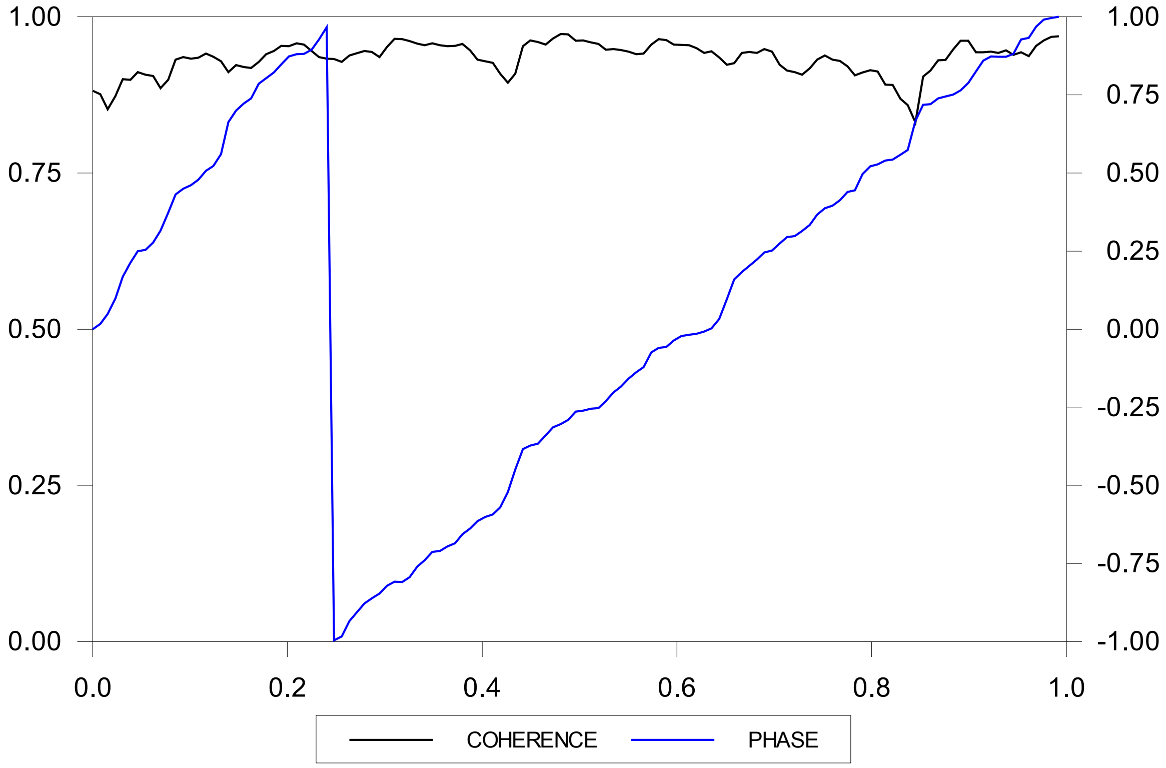
```
    Variable                          Coeff      Std Error      T-Stat      Signif
*********************************************************************************
1.  AR{1}                       0.837974838  0.095459927     8.77829  0.00000000
2.  MA{1}                      -0.609493592  0.139533318    -4.36809  0.00002354
```

Compute the cross correlations of the two differenced series. (Note that the matrix of graphs is the transpose of the format shown in the text; that is, the off-diagonal elements are switched).

```
@crosscorr dlead dsales
```




```
@crosspec dlead dsales
```




```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 7.3.1 from pp 237-239

open data e731a.tsm

data(format=free,org=columns) 1 200 x1 x2


print
```

ENTRY	X1	X2
1	-0.436230	-0.7576500
2	-0.026031	-0.6158700
3	1.223000	-0.5302700
4	1.237100	-0.6760500

5	1.103400	-0.5950900
6	0.628340	1.5365000
7	-0.482250	1.1567000
8	-0.696530	1.4978000
9	0.660250	0.9668800
10	0.399660	0.5218600
11	-0.707620	0.5111400
12	-1.539200	0.2396200
13	0.387110	0.1783900
14	0.017636	0.8342200
15	1.603200	0.9909600
16	-0.057028	3.2186000
17	-1.177300	2.3453000
18	-2.904300	2.5252000
19	-2.572900	1.0199000
20	-2.297700	0.5187800
21	-3.726100	2.2913000
22	-3.132000	2.6383000
23	-4.685300	1.5366000
24	-3.772300	1.6499000
25	-2.865000	0.5162000
26	-3.516800	1.6223000
27	-3.486700	1.2901000
28	-2.921400	-0.0502390
29	-3.192500	-1.2580000
30	-2.885400	-0.6527200
31	-3.246100	-0.1139800
32	-1.908700	-1.2726000
33	-2.700000	-0.5589300

34	-1.111500	-0.0299890
35	-0.811000	-0.1524700
36	0.519600	-0.0314800
37	0.443020	-0.6251800
38	1.188800	-1.2975000
39	1.002000	-2.5436000
40	-0.048099	-1.2367000
41	-0.142940	-1.2469000
42	-0.619680	-1.4910000
43	-0.630710	-2.4394000
44	-1.095800	-0.8968600
45	0.529090	-0.9829000
46	0.751270	-1.4428000
47	1.674400	-0.2147300
48	-0.280240	-1.7396000
49	-0.876520	-2.4244000
50	-0.267720	0.8677000
51	-0.647150	-0.6182500
52	-0.508540	-0.6390000
53	-1.751800	-0.7450500
54	-2.173300	1.4144000
55	-2.771400	0.7611500
56	-1.135700	2.8888000
57	0.337060	1.8925000
58	-0.362390	0.3764100
59	-0.233830	1.1995000
60	0.199320	-0.0402490
61	0.150970	-0.6306200
62	0.971240	-0.3253500

63	1.933400	-2.3537000
64	2.437800	-1.1711000
65	0.943620	-1.6622000
66	0.684090	-1.9835000
67	0.503470	-1.8070000
68	1.361600	-2.3381000
69	2.188600	-1.0535000
70	2.377800	-1.2183000
71	0.109020	-2.7264000
72	-0.511750	-1.0558000
73	-0.604530	-2.7261000
74	0.014211	0.5690400
75	1.852000	0.6705900
76	0.321220	-0.4434700
77	-0.233280	-0.6057700
78	0.652110	-0.4963400
79	0.565350	-0.5913600
80	-0.770810	-2.9873000
81	-0.536600	-0.4696700
82	-1.265400	0.1967600
83	-0.977470	-1.1317000
84	-0.959220	-1.0321000
85	-1.285900	1.2396000
86	-1.537300	1.6047000
87	-1.278500	-0.4295900
88	-3.016400	0.8065500
89	-1.648300	0.4760600
90	-1.389800	0.3539400
91	-1.251400	0.1468500

92	-1.234400	-2.8240000
93	0.518760	-2.2139000
94	2.515100	-2.6655000
95	0.701610	-2.3619000
96	0.994770	-1.2726000
97	0.293250	-1.2982000
98	-0.992030	-2.1642000
99	-0.510910	-1.4782000
100	-0.381860	-3.2808000
101	-0.937140	-2.3079000
102	-0.348850	-2.1251000
103	-0.374510	-1.9942000
104	-0.958910	-1.7242000
105	0.131440	-1.1759000
106	1.665400	-2.1539000
107	2.138100	-3.3359000
108	1.947200	-3.5581000
109	0.332030	-3.4665000
110	-0.145450	-3.9086000
111	-1.589300	-2.8015000
112	-0.195650	-2.6078000
113	-0.047744	-3.7381000
114	-2.193400	-3.4332000
115	-1.226000	-2.9262000
116	0.395180	-1.3145000
117	-1.174000	-1.5794000
118	-0.056431	-2.2234000
119	-0.159620	-1.4554000
120	-0.139370	-0.0514210

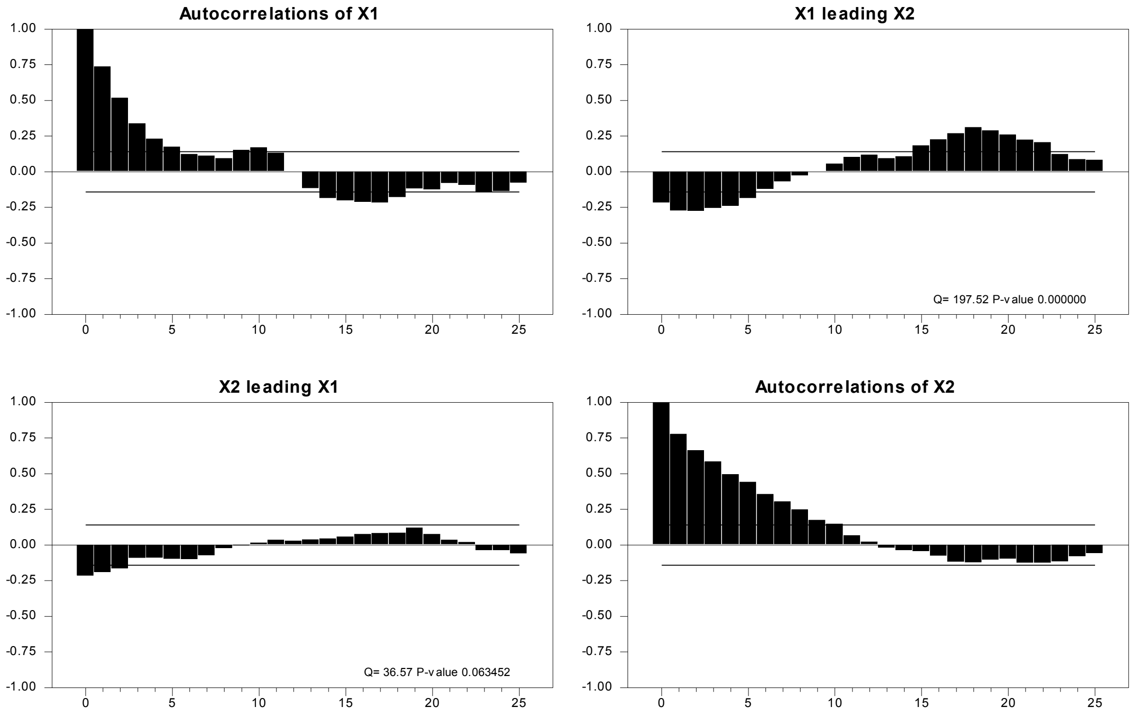
121	-1.291500	-0.0711430
122	-2.676200	-0.6007900
123	-1.995200	0.6477000
124	-2.701600	0.8557800
125	-2.947900	0.7272700
126	-2.930700	1.0237000
127	1.171700	0.7254000
128	0.925140	0.8770100
129	0.965790	0.5291300
130	0.186240	1.0876000
131	0.290300	2.3203000
132	-0.524190	2.0377000
133	-0.790430	2.4425000
134	-0.462450	1.1234000
135	-1.686600	1.1025000
136	-0.290010	0.2276700
137	0.875040	0.0551520
138	3.266100	0.3495100
139	3.104900	0.5189600
140	1.473600	-0.4070500
141	-0.037760	-1.5436000
142	0.430160	-0.0186360
143	0.048314	-0.1805700
144	0.185050	1.4535000
145	0.449140	-1.0454000
146	1.756200	0.0594060
147	3.076300	1.2203000
148	2.592100	-0.0663140
149	3.085600	0.4139400

150	0.189900	-0.5035200
151	-1.158900	-2.0622000
152	0.289670	-2.1658000
153	-1.202800	-2.5672000
154	-1.818700	-1.2070000
155	-1.384100	0.0083817
156	-0.851710	-0.3089400
157	0.817070	1.1315000
158	0.734770	0.8663900
159	1.915700	0.5564200
160	0.677670	-0.7869200
161	-0.404010	-0.2534000
162	-2.293300	1.1811000
163	-1.179900	2.1221000
164	-1.177500	3.6049000
165	-2.200300	3.1012000
166	-1.848500	3.6627000
167	-1.638700	3.1008000
168	0.533920	1.4835000
169	-0.956430	1.9641000
170	-1.630800	1.6929000
171	-2.310700	2.6392000
172	-2.059400	0.2548800
173	-0.454430	-0.1495700
174	0.944190	0.6994300
175	0.243280	0.4267800
176	0.449310	-0.0834550
177	-0.114380	0.3850900
178	0.279980	-0.1626400

179	-0.084835	0.8378100
180	-0.952590	-0.3298700
181	0.408480	-0.9971800
182	1.104100	-0.7665100
183	0.947380	-1.7806000
184	1.469100	-1.6741000
185	0.825210	-0.9367100
186	0.022989	-1.9208000
187	-1.439500	-1.6196000
188	-1.330200	-2.2071000
189	-1.930700	-0.8158900
190	0.057027	1.2856000
191	-1.650300	0.2706800
192	-1.982100	1.3084000
193	1.296100	0.4312500
194	-0.103190	0.2599300
195	0.420640	1.7020000
196	-0.195500	4.0209000
197	0.784690	3.2623000
198	1.759800	1.8206000
199	2.378800	0.1357600
200	2.937900	1.1499000

Cross correlation plots of the raw series

```
@crosscorr x1 x2
```



Estimate the AR(1) models on each, and save the residuals

```
boxjenk(ar=1,demean,maxl) x1 / rx1

boxjenk(ar=1,demean,maxl) x2 / rx2
```

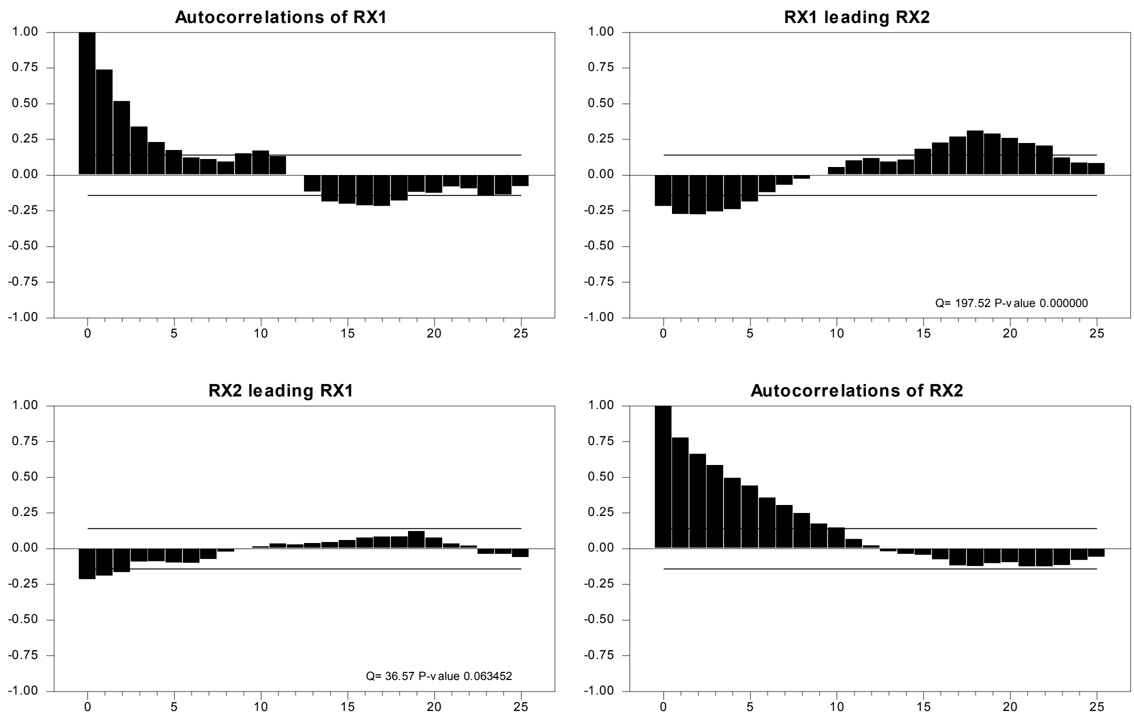
```
Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     1 Iterations. Final criterion was  0.0000000 <=  0.0000100

Dependent Variable X1

Usable Observations                        200

Degrees of Freedom                         199

Centered R^2                         0.0000000

R-Bar^2                              0.0000000

Uncentered R^2                       0.0441695

Mean of Dependent Variable        -0.325255055
```

```
Std Error of Dependent Variable   1.516847320
Standard Error of Estimate        1.516847320
Sum of Squared Residuals        457.86433278
Log Likelihood                    -366.6133
Durbin-Watson Statistic              0.5016
Q(36-1)                            310.2405
Significance Level of Q           0.0000000

    Variable                       Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                       0.8000000000 0.0000000000      0.00000  0.00000000


Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     1 Iterations. Final criterion was  0.0000000 <=  0.0000100
Dependent Variable X2
Usable Observations                     200
Degrees of Freedom                      199
Centered R^2                     -0.0000000
R-Bar^2                          -0.0000000
Uncentered R^2                    0.0162652
Mean of Dependent Variable     -0.207977132
Std Error of Dependent Variable   1.621489000
Standard Error of Estimate        1.621489000
Sum of Squared Residuals        523.21608894
Log Likelihood                    -379.9554
Durbin-Watson Statistic              0.4419
Q(36-1)                            538.7446
Significance Level of Q           0.0000000
```

```
   Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                        0.8000000000 0.0000000000      0.00000  0.00000000
```

Cross correlation plots of the residuals

```
@crosscorr rx1 rx2
```




```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 7.6.2 from pp 248-249
open data ls2.tsm
data(format=free,org=columns) 1 150 lead sales
```

```
*

print
```

ENTRY	LEAD	SALES
1	10.01	200.1
2	10.07	199.5
3	10.32	199.4
4	9.75	198.9
5	10.33	199.0
6	10.13	200.2
7	10.36	198.6
8	10.32	200.0
9	10.13	200.3
10	10.16	201.2
11	10.58	201.6
12	10.62	201.5
13	10.86	201.5
14	11.20	203.5
15	10.74	204.9
16	10.56	207.1
17	10.48	210.5
18	10.77	210.5
19	11.33	209.8
20	10.96	208.8
21	11.16	209.5
22	11.70	213.2
23	11.39	213.7
24	11.42	215.1

25	11.94	218.7
26	11.24	219.8
27	11.59	220.5
28	10.96	223.8
29	11.40	222.8
30	11.02	223.8
31	11.01	221.7
32	11.23	222.3
33	11.33	220.8
34	10.83	219.4
35	10.84	220.1
36	11.14	220.6
37	10.38	218.9
38	10.90	217.8
39	11.05	217.7
40	11.11	215.0
41	11.01	215.3
42	11.22	215.9
43	11.21	216.7
44	11.91	216.7
45	11.69	217.7
46	10.93	218.7
47	10.99	222.9
48	11.01	224.9
49	10.84	222.2
50	10.76	220.7
51	10.77	220.0
52	10.88	218.7
53	10.49	217.0

54	10.50	215.9
55	11.00	215.8
56	10.98	214.1
57	10.61	212.3
58	10.48	213.9
59	10.53	214.6
60	11.07	213.6
61	10.61	212.1
62	10.86	211.4
63	10.34	213.1
64	10.78	212.9
65	10.80	213.3
66	10.33	211.5
67	10.44	212.3
68	10.50	213.0
69	10.75	211.0
70	10.40	210.7
71	10.40	210.1
72	10.34	211.4
73	10.55	210.0
74	10.46	209.7
75	10.82	208.8
76	10.91	208.8
77	10.87	208.8
78	10.67	210.6
79	11.11	211.9
80	10.88	212.8
81	11.28	212.5
82	11.27	214.8

83	11.44	215.3
84	11.52	217.5
85	12.10	218.8
86	11.83	220.7
87	12.62	222.2
88	12.41	226.7
89	12.43	228.4
90	12.73	233.2
91	13.01	235.7
92	12.74	237.1
93	12.73	240.6
94	12.76	243.8
95	12.92	245.3
96	12.64	246.0
97	12.79	246.3
98	13.05	247.7
99	12.69	247.6
100	13.01	247.8
101	12.90	249.4
102	13.12	249.0
103	12.47	249.9
104	12.47	250.5
105	12.94	251.5
106	13.10	249.0
107	12.91	247.6
108	13.39	248.8
109	13.13	250.4
110	13.34	250.7
111	13.34	253.0

112	13.14	253.7
113	13.49	255.0
114	13.87	256.2
115	13.39	256.0
116	13.59	257.4
117	13.27	260.4
118	13.70	260.0
119	13.20	261.3
120	13.32	260.4
121	13.15	261.6
122	13.30	260.8
123	12.94	259.8
124	13.29	259.0
125	13.26	258.9
126	13.08	257.4
127	13.24	257.7
128	13.31	257.9
129	13.52	257.4
130	13.02	257.3
131	13.25	257.6
132	13.12	258.9
133	13.26	257.8
134	13.11	257.7
135	13.30	257.2
136	13.06	257.5
137	13.32	256.8
138	13.10	257.5
139	13.27	257.0
140	13.64	257.6

141	13.58	257.3
142	13.87	257.5
143	13.53	259.6
144	13.41	261.1
145	13.25	262.9
146	13.50	263.3
147	13.58	262.8
148	13.51	261.8
149	13.77	262.2
150	13.40	262.7

```
set dlead = lead-lead{1}
set dsales = sales-sales{1}

@yulevar(lags=8,crit=aic,model=test,print)
# dlead dsales

Lags    AICC
   0 614.122350
   1 571.349035
   2 535.606829
   3 220.902300
   4 162.510017
   5 158.424359*
   6 164.493385
   7 169.568491
   8 175.087931
```

Estimated VAR Equations

Dependent Variable DLEAD

	Variable	Coeff
1.	DLEAD{1}	-0.517043294
2.	DLEAD{2}	-0.191954744
3.	DLEAD{3}	-0.073329575
4.	DLEAD{4}	-0.031762570
5.	DLEAD{5}	0.021493395
6.	DSALES{1}	0.024091714
7.	DSALES{2}	-0.017620399
8.	DSALES{3}	0.010014657
9.	DSALES{4}	-0.008762502
10.	DSALES{5}	0.011381961
11.	Constant	0.032757739

Dependent Variable DSALES

	Variable	Coeff
1.	DLEAD{1}	-0.019087532
2.	DLEAD{2}	0.046839615
3.	DLEAD{3}	4.677751056
4.	DLEAD{4}	3.664357955
5.	DLEAD{5}	1.300113053
6.	DSALES{1}	-0.050631003

```
      7.   DSALES{2}          0.249682872
      8.   DSALES{3}          0.206464744
      9.   DSALES{4}          0.004438521
      10.  DSALES{5}          0.029279647
      11.  Constant           0.015588525
```

```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 7.6.3 from page 251-252
open data djaopc2.tsm
data(format=free,org=columns) 1 250 dj ao
print
```

```
 ENTRY          DJ              AO
      1     0.347357405    -0.164821015
      2    -0.507675671    -1.310426663
      3     0.494778414    -0.486172827
      4    -0.077057504    -0.052532045
      5    -0.484734979     1.203616104
      6    -1.036463018    -0.072708387
      7    -1.078360087     0.057169586
      8     0.276486752    -0.306461666
      9    -0.204960784     0.484551659
     10     0.094921958     0.928134398
     11     0.694023048    -0.179809915
     12    -0.047089161    -0.036026763
     13     0.007851891     0.468516707
     14    -0.313490172     0.666188378
     15     0.731058305     0.422520872
```

16	-0.093546414	0.217975364
17	0.265529270	1.087506323
18	0.326990516	1.015761821
19	-0.426786404	0.183277194
20	0.030974180	0.182941904
21	0.241858545	0.671207186
22	-0.007792042	-0.578488087
23	0.279978737	0.522682446
24	0.511768738	1.147846561
25	0.223656199	0.586808923
26	0.346582253	0.549635987
27	-0.191911177	-0.479501319
28	0.269027211	-1.146711636
29	-0.245260761	0.424038602
30	0.361370237	0.048534265
31	0.666155153	-0.703405453
32	-0.030487722	0.429918413
33	-0.213206843	0.612929902
34	0.633073737	0.386791084
35	-0.197133297	1.729037230
36	0.326578076	0.956348831
37	0.136218014	-0.332958169
38	-0.967373784	-0.795181857
39	-1.007408783	-0.322519446
40	0.508968325	-1.032546631
41	0.122686589	-1.235636329
42	-0.214644042	-0.165514556
43	0.645042540	-0.385215526
44	-0.030571440	0.465025209

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47	0.904141922	0.043208987
48	-0.173009914	0.983779633
49	-0.513180450	0.175830442
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51	-0.643203456	-1.627304147
52	0.106804714	-1.937246865
53	0.364980390	1.134554140
54	0.000000000	0.472347963
55	-0.098438542	0.053868756
56	-0.166940376	-1.605403553
57	0.167219533	-0.044769437
58	0.356410918	-0.199064397
59	0.136053318	2.089358731
60	0.052942781	0.004884482
61	0.165763606	0.307707336
62	0.233679495	0.983590593
63	0.420825625	1.065625151
64	-0.127191373	-0.014312977
65	0.291974326	-0.515340936
66	0.635180329	0.460453739
67	-0.579104938	-0.377178324
68	-0.686950086	-0.800345059
69	0.248054841	0.236726412
70	0.682475699	0.260266050
71	0.097026045	-0.201903663
72	-0.267894472	-0.433526012
73	0.454988452	-0.667634253

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77	0.022146467	0.000000000
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79	-0.486251855	1.297663266
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82	0.726720971	0.032204637
83	0.394302175	0.878443637
84	0.133199257	0.314580104
85	0.444020316	-0.649911376
86	1.170968156	0.576395242
87	-0.393221076	0.368416265
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89	-0.161096286	-0.824376025
90	0.644644144	1.327210103
91	0.079902772	1.441261784
92	0.000000000	-0.272540434
93	0.363797028	0.721293849
94	0.195398482	0.800640512
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96	-0.043173040	-1.146513798
97	-0.445973334	-0.116880198
98	0.325003722	0.000000000
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103	0.290867077	0.086576339
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113	0.229108488	0.671441361
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124	-0.191035926	-0.120705664
125	0.206044315	-1.608255090
126	0.624168254	1.318027211
127	-0.116694587	1.258917331
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143	-1.953760726	-0.399847677
144	0.253946371	-1.878225961
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150	-0.514450648	-0.258695027
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154	0.048614354	-0.767336112
155	-0.048590732	-0.687350835
156	-1.121134408	0.735364799
157	-0.016572663	-1.646070900
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159	1.495813778	-0.830645949
160	-0.105679883	0.635593220

161	1.564949516	0.000000000
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166	0.525030625	-0.880886695
167	0.361792155	-0.180672884
168	-0.448523453	-1.262107426
169	-0.048137381	-1.501189061
170	-0.716185467	0.814848348
171	-1.102602534	0.249463653
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173	-0.748548440	1.314610099
174	0.655820823	0.334218028
175	0.187251563	1.400999314
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177	1.337600436	-0.705860060
178	0.330053405	0.562977099
179	0.698922283	-0.668943923
180	0.196063826	0.487175813
181	-0.635628659	0.812776273
182	0.073749269	0.537482320
183	0.270481714	-1.242731195
184	-0.048997417	-0.427370720
185	0.098042872	0.252754066
186	0.000000000	-0.494719817
187	0.032737667	-0.478057176
188	0.065453907	0.739744452
189	-0.048925370	-0.963189014

190	0.351956244	0.077034184
191	-0.098085478	-0.293466756
192	-0.334614119	-0.110977081
193	-0.171995602	0.458892861
194	0.098414434	-0.144251575
195	0.541146880	-0.351519237
196	0.256264161	0.000000000
197	0.838197044	0.347926935
198	-0.640133374	-0.105942406
199	0.552183009	-0.887003471
200	-0.906767698	-0.233463035
201	-0.923538040	-1.306552262
202	-0.906758599	-1.521438451
203	0.453078099	0.867776886
204	-0.689438543	0.576856134
205	-1.680142954	-0.227441286
206	1.335188373	-2.998166411
207	-0.430335097	0.868499029
208	-0.070579130	0.035453809
209	-1.147789095	0.708824870
210	0.598351430	-1.171384043
211	0.000000000	1.083528335
212	0.159872760	0.820290876
213	0.602878045	-0.608964760
214	0.378827051	-1.456408196
215	0.561758151	0.137600652
216	-0.165806629	-0.188304748
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219	0.944043107	1.470736885
220	0.389382898	2.505354386
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222	-0.189592137	0.246103363
223	-0.561319635	0.057764513
224	0.138975711	-1.414413548
225	0.069391419	0.161038454
226	0.182059630	-0.204628502
227	-0.164624890	-0.341746814
228	-0.407154788	0.019595356
229	0.278459442	0.019591517
230	0.902480145	0.950002448
231	0.894408288	0.999272374
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233	-0.106948491	-0.699602281
234	-0.695382051	0.540462288
235	-0.498434591	0.403167747
236	0.181210669	-0.516277069
237	0.051947222	0.273893614
238	0.292883464	-0.493578685
239	-0.421051514	-0.659764026
240	0.474819377	-0.528407989
241	-0.223418623	0.185194210
242	0.645694880	-0.763730116
243	-0.213762726	0.955882353
244	-0.557397365	0.354454965
245	-0.008520995	-0.266111864
246	-0.103592172	0.111580071
247	0.656053231	-0.581508044

```
    248     1.877732843     0.463053227

    249    -0.437774421     0.810246956

    250     1.335808600    -0.028876697
```

```
@YuleVAR(lags=1,model=multiv)
# dj ao
diff(center) ao / cao
@bjautofit(pmax=5,qmax=0) cao
boxjenk(ar=1,maxl,constant,define=univ) ao
boxjenk(ar=0,maxl,constant,define=meanonly) ao
```

```
Estimated VAR Equations



Dependent Variable DJ
     Variable            Coeff
***********************************
1.   DJ{1}            -0.014837964
2.   AO{1}             0.035731895
3.   Constant          0.028843834



Dependent Variable AO
     Variable            Coeff
***********************************
1.   DJ{1}            0.6588983152
2.   AO{1}            0.0997626799
3.   Constant         0.0083626346
```

```
AIC analysis of models for series CAO

   MA

AR   0

 0 644.5251

 1 646.5251

 2 643.4294*

 3 644.7642

 4 646.3510

 5 646.8574
```

```
Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     2 Iterations. Final criterion was  0.0000000 <=  0.0000100

Dependent Variable AO

Usable Observations                      250

Degrees of Freedom                       248

Centered R^2                       0.0000000

R-Bar^2                           -0.0040323

Uncentered R^2                     0.0012356

Mean of Dependent Variable      0.0308879634

Std Error of Dependent Variable  0.8799551968

Standard Error of Estimate      0.8817275152

Sum of Squared Residuals        192.80596596

Regression F(1,248)              2.2027e-013

Significance Level of F            0.9999996

Log Likelihood                     -322.2626

Durbin-Watson Statistic               1.7626

Q(36-1)                              33.2796
```

```
Significance Level of Q             0.5513120

    Variable                           Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  CONSTANT                       0.0308879634 0.0557653445      0.55389  0.58015200
2.  AR{1}                          0.8000000000 0.0000000000      0.00000  0.00000000


Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     2 Iterations. Final criterion was 0.0000000 <=  0.0000100
Dependent Variable AO
Usable Observations                     250
Degrees of Freedom                      249
Centered R^2                      0.0000000
R-Bar^2                           0.0000000
Uncentered R^2                    0.0012356
Mean of Dependent Variable     0.0308879634
Std Error of Dependent Variable 0.8799551968
Standard Error of Estimate     0.8799551968
Sum of Squared Residuals       192.80596596
Log Likelihood                    -322.2626
Durbin-Watson Statistic              1.7626
Q(36-0)                             33.2796
Significance Level of Q           0.5986594

    Variable                           Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  CONSTANT                       0.0308879634 0.0556532532      0.55501 0.57938787
```

```
open data djaopcf2.tsm

data(format=free,org=columns) 251 290 dj ao


print
```

ENTRY	DJ	AO	COVARS	CAO
1	0.347357405	-0.164821015	0.771223863836	-0.195708978396
2	-0.507675671	-1.310426663	0.091463864451	-1.341314626396
3	0.494778414	-0.486172827	NA	-0.517060790396
4	-0.077057504	-0.052532045	NA	-0.083420008396
5	-0.484734979	1.203616104	NA	1.172728140604
6	-1.036463018	-0.072708387	NA	-0.103596350396
7	-1.078360087	0.057169586	NA	0.026281622604
8	0.276486752	-0.306461666	NA	-0.337349629396
9	-0.204960784	0.484551659	NA	0.453663695604
10	0.094921958	0.928134398	NA	0.897246434604
11	0.694023048	-0.179809915	NA	-0.210697878396
12	-0.047089161	-0.036026763	NA	-0.066914726396
13	0.007851891	0.468516707	NA	0.437628743604
14	-0.313490172	0.666188378	NA	0.635300414604
15	0.731058305	0.422520872	NA	0.391632908604
16	-0.093546414	0.217975364	NA	0.187087400604
17	0.265529270	1.087506323	NA	1.056618359604
18	0.326990516	1.015761821	NA	0.984873857604
19	-0.426786404	0.183277194	NA	0.152389230604
20	0.030974180	0.182941904	NA	0.152053940604
21	0.241858545	0.671207186	NA	0.640319222604
22	-0.007792042	-0.578488087	NA	-0.609376050396
23	0.279978737	0.522682446	NA	0.491794482604

24	0.511768738	1.147846561	NA	1.116958597604
25	0.223656199	0.586808923	NA	0.555920959604
26	0.346582253	0.549635987	NA	0.518748023604
27	-0.191911177	-0.479501319	NA	-0.510389282396
28	0.269027211	-1.146711636	NA	-1.177599599396
29	-0.245260761	0.424038602	NA	0.393150638604
30	0.361370237	0.048534265	NA	0.017646301604
31	0.666155153	-0.703405453	NA	-0.734293416396
32	-0.030487722	0.429918413	NA	0.399030449604
33	-0.213206843	0.612929902	NA	0.582041938604
34	0.633073737	0.386791084	NA	0.355903120604
35	-0.197133297	1.729037230	NA	1.698149266604
36	0.326578076	0.956348831	NA	0.925460867604
37	0.136218014	-0.332958169	NA	-0.363846132396
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57	0.167219533	-0.044769437	NA	-0.075657400396
58	0.356410918	-0.199064397	NA	-0.229952360396
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61	0.165763606	0.307707336	NA	0.276819372604
62	0.233679495	0.983590593	NA	0.952702629604
63	0.420825625	1.065625151	NA	1.034737187604
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68	-0.686950086	-0.800345059	NA	-0.831233022396
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72	-0.267894472	-0.433526012	NA	-0.464413975396
73	0.454988452	-0.667634253	NA	-0.698522216396
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75	0.000000000	0.986078886	NA	0.955190922604
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105	-2.425611065	-0.333247885	NA	-0.364135848396
106	0.901478011	-2.216220850	NA	-2.247108813396
107	-0.007423867	1.065275525	NA	1.034387561604
108	0.662821330	-1.496486510	NA	-1.527374473396
109	-0.930334290	-1.611695803	NA	-1.642583766396
110	-0.014376152	0.317772904	NA	0.286884940604

111	0.238267630	-0.124921924	NA	-0.155809887396
112	0.620123666	-0.205485571	NA	-0.236373534396
113	0.229108488	0.671441361	NA	0.640553397604
114	-0.371577260	-0.360160071	NA	-0.391048034396
115	-0.896844982	-0.789861216	NA	-0.820749179396
116	0.000000000	-2.010615329	NA	-2.041503292396
117	0.622514444	1.101675465	NA	1.070787501604
118	-0.510780589	0.744608400	NA	0.713720436604
119	-1.330530773	-0.986975528	NA	-1.017863491396
120	-0.029167426	-2.193900774	NA	-2.224788737396
121	-0.176097614	1.456626955	NA	1.425738991604
122	-0.594725497	0.073391129	NA	0.042503165604
123	0.590933076	-1.269652106	NA	-1.300540069396
124	-0.191035926	-0.120705664	NA	-0.151593627396
125	0.206044315	-1.608255090	NA	-1.639143053396
126	0.624168254	1.318027211	NA	1.287139247604
127	-0.116694587	1.258917331	NA	1.228029367604
128	0.043876502	-1.146567205	NA	-1.177455168396
129	-0.591424219	0.386621949	NA	0.355733985604
130	0.837462343	-0.092803118	NA	-0.123691081396
131	0.007248816	1.216850123	NA	1.185962159604
132	-0.346623591	-0.312026798	NA	-0.342914761396
133	-0.037406581	0.046029919	NA	0.015141955604
134	0.441510856	-0.418679549	NA	-0.449567512396
135	0.789363387	-0.041581963	NA	-0.072469926396
136	-0.790625441	-1.063092212	NA	-1.093980175396
137	-0.059510718	0.014015417	NA	-0.016872546396
138	0.178897361	1.873131540	NA	1.842243576604
139	-1.250045226	-0.508964189	NA	-0.539852152396

140	-1.213266372	-0.838786985	NA	-0.869674948396
141	-0.327970477	-1.984569623	NA	-2.015457586396
142	-1.683256475	-0.384086491	NA	-0.414974454396
143	-1.953760726	-0.399847677	NA	-0.430735640396
144	0.253946371	-1.878225961	NA	-1.909113924396
145	0.000000000	0.000000000	NA	-0.030887963396
146	-1.171905081	0.000000000	NA	-0.030887963396
147	2.283662877	-0.150991184	NA	-0.181879147396
148	0.117537907	1.663414634	NA	1.632526670604
149	0.367690021	0.158341730	NA	0.127453766604
150	-0.514450648	-0.258695027	NA	-0.289582990396
151	0.396542433	-0.288184438	NA	-0.319072401396
152	-0.193557307	0.920038536	NA	0.889150572604
153	-0.549204306	0.768459739	NA	0.737571775604
154	0.048614354	-0.767336112	NA	-0.798224075396
155	-0.048590732	-0.687350835	NA	-0.718238798396
156	-1.121134408	0.735364799	NA	0.704476835604
157	-0.016572663	-1.646070900	NA	-1.676958863396
158	-0.583178169	-0.717958669	NA	-0.748846632396
159	1.495813778	-0.830645949	NA	-0.861533912396
160	-0.105679883	0.635593220	NA	0.604705256604
161	1.564949516	0.000000000	NA	-0.030887963396
162	-0.168385603	1.317013464	NA	1.286125500604
163	0.000000000	-0.468734899	NA	-0.499622862396
164	-0.844159004	0.456377142	NA	0.425489178604
165	0.364745619	-0.144990576	NA	-0.175878539396
166	0.525030625	-0.880886695	NA	-0.911774658396
167	0.361792155	-0.180672884	NA	-0.211560847396
168	-0.448523453	-1.262107426	NA	-1.292995389396

169	-0.048137381	-1.501189061	NA	-1.532077024396
170	-0.716185467	0.814848348	NA	0.783960384604
171	-1.102602534	0.249463653	NA	0.218575689604
172	0.754193947	-0.054745434	NA	-0.085633397396
173	-0.748548440	1.314610099	NA	1.283722135604
174	0.655820823	0.334218028	NA	0.303330064604
175	0.187251563	1.400999314	NA	1.370111350604
176	0.322979058	1.975845411	NA	1.944957447604
177	1.337600436	-0.705860060	NA	-0.736748023396
178	0.330053405	0.562977099	NA	0.532089135604
179	0.698922283	-0.668943923	NA	-0.699831886396
180	0.196063826	0.487175813	NA	0.456287849604
181	-0.635628659	0.812776273	NA	0.781888309604
182	0.073749269	0.537482320	NA	0.506594356604
183	0.270481714	-1.242731195	NA	-1.273619158396
184	-0.048997417	-0.427370720	NA	-0.458258683396
185	0.098042872	0.252754066	NA	0.221866102604
186	0.000000000	-0.494719817	NA	-0.525607780396
187	0.032737667	-0.478057176	NA	-0.508945139396
188	0.065453907	0.739744452	NA	0.708856488604
189	-0.048925370	-0.963189014	NA	-0.994076977396
190	0.351956244	0.077034184	NA	0.046146220604
191	-0.098085478	-0.293466756	NA	-0.324354719396
192	-0.334614119	-0.110977081	NA	-0.141865044396
193	-0.171995602	0.458892861	NA	0.428004897604
194	0.098414434	-0.144251575	NA	-0.175139538396
195	0.541146880	-0.351519237	NA	-0.382407200396
196	0.256264161	0.000000000	NA	-0.030887963396
197	0.838197044	0.347926935	NA	0.317038971604

198	-0.640133374	-0.105942406	NA	-0.136830369396
199	0.552183009	-0.887003471	NA	-0.917891434396
200	-0.906767698	-0.233463035	NA	-0.264350998396
201	-0.923538040	-1.306552262	NA	-1.337440225396
202	-0.906758599	-1.521438451	NA	-1.552326414396
203	0.453078099	0.867776886	NA	0.836888922604
204	-0.689438543	0.576856134	NA	0.545968170604
205	-1.680142954	-0.227441286	NA	-0.258329249396
206	1.335188373	-2.998166411	NA	-3.029054374396
207	-0.430335097	0.868499029	NA	0.837611065604
208	-0.070579130	0.035453809	NA	0.004565845604
209	-1.147789095	0.708824870	NA	0.677936906604
210	0.598351430	-1.171384043	NA	-1.202272006396
211	0.000000000	1.083528335	NA	1.052640371604
212	0.159872760	0.820290876	NA	0.789402912604
213	0.602878045	-0.608964760	NA	-0.639852723396
214	0.378827051	-1.456408196	NA	-1.487296159396
215	0.561758151	0.137600652	NA	0.106712688604
216	-0.165806629	-0.188304748	NA	-0.219192711396
217	-0.008911717	0.596573526	NA	0.565685562604
218	0.043752329	0.288914795	NA	0.258026831604
219	0.944043107	1.470736885	NA	1.439848921604
220	0.389382898	2.505354386	NA	2.474466422604
221	0.043156153	0.694849368	NA	0.663961404604
222	-0.189592137	0.246103363	NA	0.215215399604
223	-0.561319635	0.057764513	NA	0.026876549604
224	0.138975711	-1.414413548	NA	-1.445301511396
225	0.069391419	0.161038454	NA	0.130150490604
226	0.182059630	-0.204628502	NA	-0.235516465396

227	-0.164624890	-0.341746814	NA	-0.372634777396
228	-0.407154788	0.019595356	NA	-0.011292607396
229	0.278459442	0.019591517	NA	-0.011296446396
230	0.902480145	0.950002448	NA	0.919114484604
231	0.894408288	0.999272374	NA	0.968384410604
232	-0.051340514	0.230536478	NA	0.199648514604
233	-0.106948491	-0.699602281	NA	-0.730490244396
234	-0.695382051	0.540462288	NA	0.509574324604
235	-0.498434591	0.403167747	NA	0.372279783604
236	0.181210669	-0.516277069	NA	-0.547165032396
237	0.051947222	0.273893614	NA	0.243005650604
238	0.292883464	-0.493578685	NA	-0.524466648396
239	-0.421051514	-0.659764026	NA	-0.690651989396
240	0.474819377	-0.528407989	NA	-0.559295952396
241	-0.223418623	0.185194210	NA	0.154306246604
242	0.645694880	-0.763730116	NA	-0.794618079396
243	-0.213762726	0.955882353	NA	0.924994389604
244	-0.557397365	0.354454965	NA	0.323567001604
245	-0.008520995	-0.266111864	NA	-0.296999827396
246	-0.103592172	0.111580071	NA	0.080692107604
247	0.656053231	-0.581508044	NA	-0.612396007396
248	1.877732843	0.463053227	NA	0.432165263604
249	-0.437774421	0.810246956	NA	0.779358992604
250	1.335808600	-0.028876697	NA	-0.059764660396
251	0.458600000	1.665800000	NA	NA
252	0.473200000	0.222500000	NA	NA
253	-0.099000000	0.264600000	NA	NA
254	-0.306100000	-0.782200000	NA	NA
255	-0.406500000	0.071200000	NA	NA

256	0.000000000	-0.545800000	NA	NA
257	0.337700000	0.386500000	NA	NA
258	-0.319300000	0.038000000	NA	NA
259	0.571500000	-0.727000000	NA	NA
260	-0.861000000	-0.885500000	NA	NA
261	-0.373400000	-1.820700000	NA	NA
262	0.505700000	0.526300000	NA	NA
263	0.398700000	0.337600000	NA	NA
264	1.503100000	0.014700000	NA	NA
265	-0.519200000	0.399800000	NA	NA
266	0.085700000	-0.480800000	NA	NA
267	-1.717900000	-0.195200000	NA	NA
268	-0.452000000	-0.904700000	NA	NA
269	-0.375700000	0.079000000	NA	NA
270	-0.140200000	-0.024700000	NA	NA
271	0.456400000	0.113400000	NA	NA
272	0.358500000	-0.798000000	NA	NA
273	0.391900000	0.019900000	NA	NA
274	-0.607200000	0.814200000	NA	NA
275	-0.296800000	-0.093600000	NA	NA
276	0.096300000	0.108400000	NA	NA
277	-1.189500000	-1.619900000	NA	NA
278	-0.362800000	-0.911000000	NA	NA
279	-0.311000000	-0.176700000	NA	NA
280	0.579300000	-0.445300000	NA	NA
281	0.629100000	1.041900000	NA	NA
282	1.452600000	0.784800000	NA	NA
283	-0.043300000	-0.049900000	NA	NA
284	0.381900000	-0.184800000	NA	NA

```
285     0.527500000     0.355200000        NA              NA
286     0.344200000     0.408800000        NA              NA
287    -0.162800000    -0.536200000        NA              NA
288     0.472200000     0.499100000        NA              NA
289    -0.632400000     0.144000000        NA              NA
290    -0.507500000     0.907600000        NA              NA
```

```
forecast(static,steps=40,model=meanonly,results=meanfore)
forecast(static,steps=40,model=multiv,results=multifore)
forecast(static,steps=40,model=univ,results=unifore)
@uforeerrors(title="Forecasts using Mean Only") ao meanfore(1) 251 290
@uforeerrors(title="Forecasts from Univariate Model") ao unifore(1) 251 290
@uforeerrors(title="Forecasts from Bivariate Model") ao multifore(2) 251 290
```

```
Forecasts using Mean Only
From 251 to 290
Mean Error             -0.0803555
Mean Absolute Error     0.5079025
Root Mean Square Error  0.6859871
Mean Square Error        0.470578
Theil's U                0.781130
```

```
Forecasts from Univariate Model
From 251 to 290
Mean Error             0.00265844
Mean Absolute Error    0.64722391
Root Mean Square Error 0.79929301
```

```
Mean Square Error        0.638869
Theil's U                0.910151


Forecasts from Bivariate Model
From 251 to 290
Mean Error             -0.0865322
Mean Absolute Error     0.4805896
Root Mean Square Error  0.6294846
Mean Square Error        0.396251
Theil's U                0.716791

Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 8.5.2 from pp 281-283


cal(m) 1949
open data airpass.dat
data(format=free) 1949:1 1960:12 airpass

graph(footer="Figure 8-3 International airline passengers; monthly totals")
# airpass
```

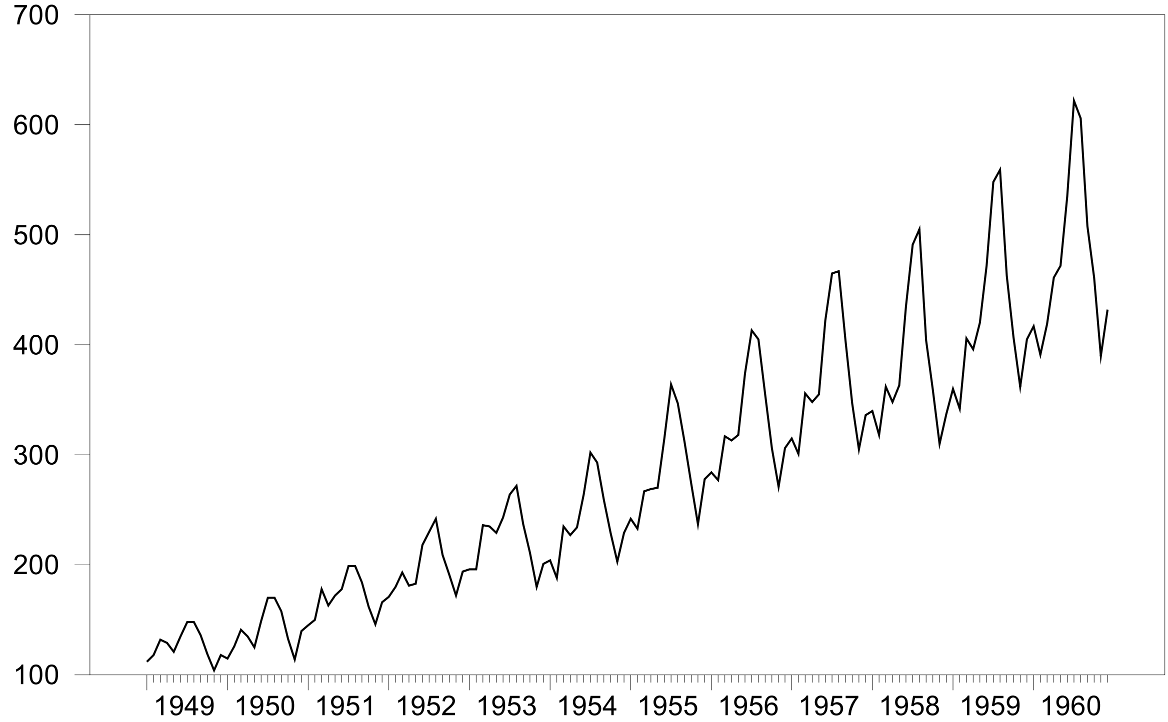


Figure 8-3 International airline passengers; monthly totals

```
The model is y(t)=c(t)+s(t)+e(t) where the
* trend term c(t) is given by:
c(t)=c(t-1)+tau(t-1)+eps1(t),
*  tau(t)=tau(t-1)+eps2(t)

and the seasonal s(t) by s(t)+s(t-1)+...+s(t-11)=eps3

The A and C matrices are fixed. We use the LocalDLM and SeasonalDLM
* procedures to create the trend and seasonal parts and combine them.

@LocalDLM(type=trend,a=at,c=ct,f=ft,shocks=both)
@SeasonalDLM(type=additive,a=as,c=cs,f=fs)
compute a=at~\as
```

```
compute c=ct~~cs

compute f=ft~\fs

The variance matrix for the shocks to the state equation is created as

 a FRML so the variances can be changed with the parameters


dec real lsigsq1 lsigsq2 lsigsq3 lsigsqv

dec frml[symm] sw

frml sw = %diag(||exp(lsigsq1),exp(lsigsq2),exp(lsigsq3)||)


Rather than estimate initial conditions, we use presample=diffuse to

* allow for a diffuse prior. As is typical of models like this, freely

* estimating the component variances tends to produce an overly

* optimistic "fit" by having the trend term adjust to fit the data

* almost perfectly, while forcing the observation equation variance to

* zero.

@LocalDLM(type=trend,a=at,c=ct,f=ft,shocks=both)

@SeasonalDLM(type=additive,a=as,c=cs,f=fs)

compute a=at~\as

compute c=ct~~cs

compute f=ft~\fs


dec real lsigsq1 lsigsq2 lsigsq3 lsigsqv

dec frml[symm] sw

frml sw = %diag(||exp(lsigsq1),exp(lsigsq2),exp(lsigsq3)||)


compute lsigsqv=log(.0001),lsigsq1=log(10.0),lsigsq2=-30.0,lsigsq3=log(10.0)

nonlin lsigsq1 lsigsq2 lsigsq3 lsigsqv lsigsq2=-30.0

dlm(a=a,c=c,f=f,sw=sw,sv=exp(lsigsqv),y=airpass,$

  presample=diffuse,pmethod=simplex,piters=10,method=bfgs,yhat=yhat) / xstates
```

```
disp "Estimated Variances"
disp %exp(%beta)
```

```
DLM - Estimation by BFGS
Convergence in    32 Iterations. Final criterion was  0.0000000 <=  0.0000100
Monthly Data From 1949:01 To 1960:12
Usable Observations                      144
Rank of Observables                      131
Log Likelihood                     -571.0140

    Variable                         Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  LSIGSQ1                        5.084940     0.116773     43.54565  0.00000000
2.  LSIGSQ2                      -30.000000     0.000000      0.00000  0.00000000
3.  LSIGSQ3                        2.935768     0.021680    135.41114  0.00000000
4.  LSIGSQV                      -24.152516  4065.050664     -0.00594  0.99525939

Estimated Variances
    161.57025  9.35762e-014      18.83597  3.24112e-011
```

```
These are the components off the KF with estimated parameters.
* xstates(t) is the state vector at time t, so we want to pull out the
* 1st and 3rd components of it. The second component is the slope
* component, which will be (almost) constant across the data set, since
* its variance is estimated at zero.
set seasonal = xstates(t)(3)
set slope    = xstates(t)(2)
set trend    = xstates(t)(1)
```

```
graph(footer="Figure 8-4 State components (contemporaneous)",$
  key=upleft) 3
# seasonal
# trend
# slope
```

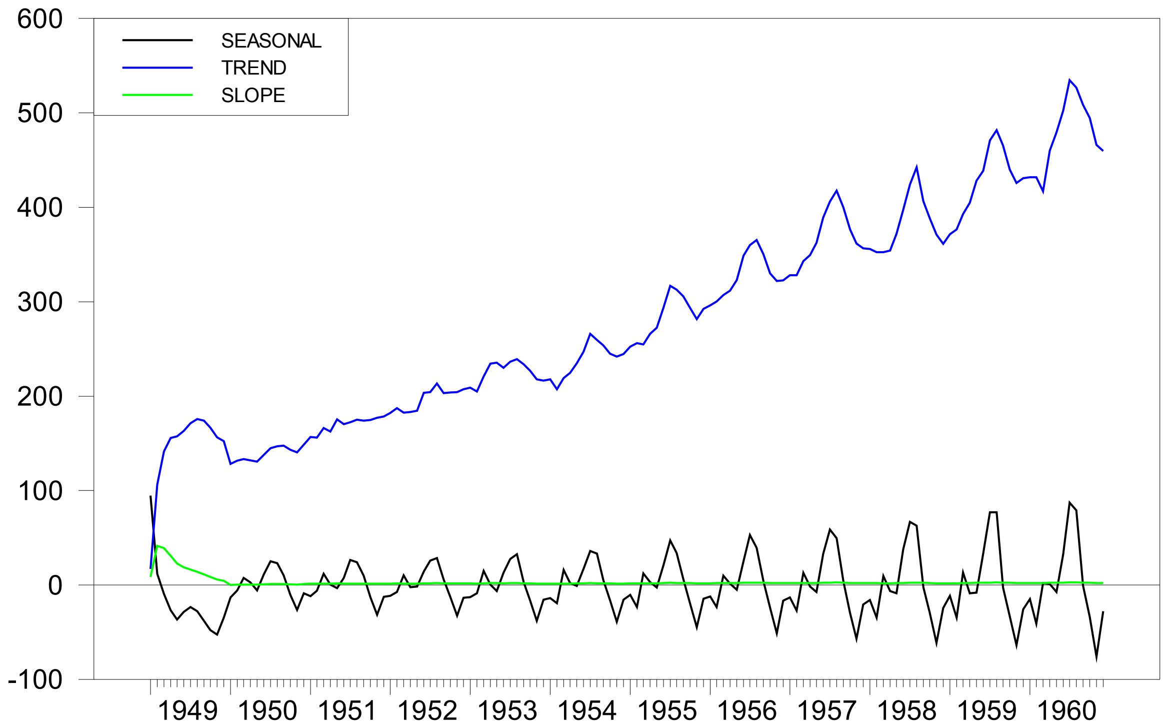


Figure 8-4 State components (contemporaneous)

```
set onestep = yhat(t)(1)
graph(footer="Figure 8-5 One-step predictors and actual data",$
  overlay=dots,ovsame) 2
# onestep
# airpass
```

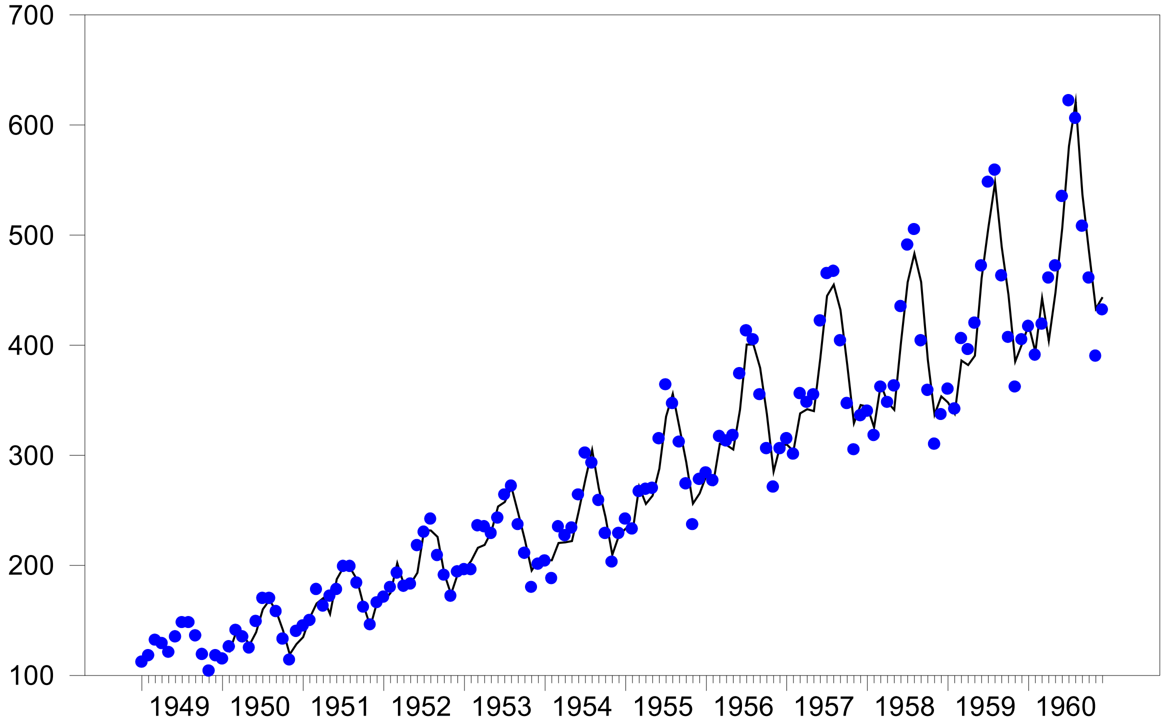


Figure 8-5 One-step predictors and actual data

The initial state vector (mu in the book) can be obtained by Kalman smoothing with the presample=diffuse option. This is much simpler than trying to add it to the parameter set. By feeding in the estimated variances from the book, we can reproduce their mu. The result found above has a higher likelihood.

```
compute lsigsq1=log(170.63),lsigsq2=-50.00,lsigsq3=log(11.338)

compute lsigsqv=log(.014179)

dlm(a=a,c=c,f=f,sw=sw,sv=exp(lsigsqv),y=airpass,$

presample=diffuse,type=smooth) / xstates

compute x1s=xstates(1)
```

Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

```
* Example 8.7.1 from pp 291-292
```

```
open data lake.dat

cal 1875

data(format=free,org=columns) 1875:1 1972:1 lake


boxjenk(ar=2,demean,maxl) lake


Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     5 Iterations. Final criterion was  0.0000067 <=  0.0000100

Dependent Variable LAKE

Annual Data From 1875:01 To 1972:01

Usable Observations                        98

Degrees of Freedom                         96

Centered R^2                        0.7215972

R-Bar^2                             0.7186971

Uncentered R^2                      0.9942157

Mean of Dependent Variable       9.0040816327

Std Error of Dependent Variable  1.3182985260

Standard Error of Estimate       0.6991990748

Sum of Squared Residuals         46.932417231

Log Likelihood                      -103.6417

Durbin-Watson Statistic                1.9277

Q(24-2)                               13.5554

Significance Level of Q             0.9165061


    Variable                        Coeff      Std Error      T-Stat      Signif

************************************************************************************

1.  AR{1}                         1.044135203  0.097996362     10.65484  0.00000000

2.  AR{2}                        -0.250268320  0.098313670     -2.54561  0.01250096
```

This generates the dummy variable for the observations we want skipped.

```
set miss = t==17.or.t==24.or.t==31.or.t==38.or.t==45.or.t==52.or.$

 t==59.or.t==66.or.t==73.or.t==80

diff(center) lake / clake
```

* This does four iterations on the EM algorithm. The %IF(x,y,z) function returns y if the expression x evaluates to a non-zero, and z if it evaluates to zero.

```
set eseries = %if(miss,0.0,clake)

do iters=1,4

   boxjenk(ar=2,maxl,noprint) eseries

disp iters eseries(17) eseries(24) eseries(31) %beta(1) %beta(2) -2*%logl

   compute v0=1+%beta(1)^2+%beta(2)^2,v2=%beta(2)/v0,v1=(%beta(1)-
%beta(1)*%beta(2))/v0

   set eseries = %if(miss,v2*(clake{2}+clake{-2})+v1*(clake{1}+clake{-1}),clake)

end do iters
```

```
Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     5 Iterations. Final criterion was  0.0000022 <=  0.0000100

Dependent Variable ESERIES

Annual Data From 1875:01 To 1972:01

Usable Observations                      98

Degrees of Freedom                       96

Centered R^2                      0.5558748

R-Bar^2                           0.5512485

Uncentered R^2                    0.5561859
```

```
Mean of Dependent Variable       0.0332736360
Std Error of Dependent Variable  1.2631355839
Standard Error of Estimate       0.8461608833
Sum of Squared Residuals         68.734871085
Log Likelihood                      -122.0832
Durbin-Watson Statistic                1.9831
Q(24-2)                               20.7451
Significance Level of Q             0.5365343

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                        0.7020423556 0.1015806471      6.91118  0.00000000
2.  AR{2}                        0.0575550432 0.1017875395      0.56544  0.57309172


Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     5 Iterations. Final criterion was  0.0000022 <=  0.0000100
Dependent Variable ESERIES
Annual Data From 1875:01 To 1972:01
Usable Observations                        98
Degrees of Freedom                         96
Centered R^2                        0.5558748
R-Bar^2                             0.5512485
Uncentered R^2                      0.5561859
Mean of Dependent Variable       0.0332736360
Std Error of Dependent Variable  1.2631355839
Standard Error of Estimate       0.8461608833
Sum of Squared Residuals         68.734871085
Log Likelihood                      -122.0832
```

```
Durbin-Watson Statistic              1.9831

Q(24-2)                             20.7451

Significance Level of Q           0.5365343


    Variable                         Coeff      Std Error      T-Stat      Signif

************************************************************************************

1.  AR{1}                        0.7020423556 0.1015806471      6.91118  0.00000000

2.  AR{2}                        0.0575550432 0.1017875395      0.56544  0.57309172


1       0.00000       0.00000       0.00000       0.70204       0.05756     244.16638


Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     5 Iterations. Final criterion was  0.0000071 <=  0.0000100

Dependent Variable ESERIES

Annual Data From 1875:01 To 1972:01

Usable Observations                     98

Degrees of Freedom                      96

Centered R^2                      0.7225577

R-Bar^2                           0.7196677

Uncentered R^2                    0.7225789

Mean of Dependent Variable     0.0113777206

Std Error of Dependent Variable  1.3092298864

Standard Error of Estimate     0.6931903321

Sum of Squared Residuals       46.129232308

Log Likelihood                    -102.7937

Durbin-Watson Statistic              1.9308

Q(24-2)                             13.4580

Significance Level of Q           0.9195960
```

```
   Variable                        Coeff      Std Error      T-Stat      Signif
*******************************************************************************
1.  AR{1}                        1.033404317  0.098417598     10.50020  0.00000000
2.  AR{2}                       -0.234200915  0.098742148     -2.37184  0.01969517
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     5 Iterations. Final criterion was  0.0000071 <=  0.0000100
Dependent Variable ESERIES
Annual Data From 1875:01 To 1972:01
Usable Observations                     98
Degrees of Freedom                      96
Centered R^2                     0.7225577
R-Bar^2                          0.7196677
Uncentered R^2                   0.7225789
Mean of Dependent Variable    0.0113777206
Std Error of Dependent Variable 1.3092298864
Standard Error of Estimate    0.6931903321
Sum of Squared Residuals      46.129232308
Log Likelihood                   -102.7937
Durbin-Watson Statistic             1.9308
Q(24-2)                            13.4580
Significance Level of Q          0.9195960
```


```
   Variable                        Coeff      Std Error      T-Stat      Signif
*******************************************************************************
1.  AR{1}                        1.033404317  0.098417598     10.50020  0.00000000
2.  AR{2}                       -0.234200915  0.098742148     -2.37184  0.01969517
```

2	0.52926	0.15891	0.70248	1.03340	-0.23420	205.58734

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in 5 Iterations. Final criterion was 0.0000072 <= 0.0000100

Dependent Variable ESERIES

Annual Data From 1875:01 To 1972:01

Usable Observations	98
Degrees of Freedom	96
Centered R^2	0.7312754
R-Bar^2	0.7284761
Uncentered R^2	0.7312941
Mean of Dependent Variable	0.0109763389
Std Error of Dependent Variable	1.3210958382
Standard Error of Estimate	0.6883959762
Sum of Squared Residuals	45.493345926
Log Likelihood	-102.1373
Durbin-Watson Statistic	1.9183
Q(24-2)	12.3846
Significance Level of Q	0.9489196

Variable	Coeff	Std Error	T-Stat	Signif
1. AR{1}	1.062995267	0.097536434	10.89844	0.00000000
2. AR{2}	-0.265837108	0.097852464	-2.71671	0.00782164

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in 5 Iterations. Final criterion was 0.0000072 <= 0.0000100

Dependent Variable ESERIES

Annual Data From 1875:01 To 1972:01

Usable Observations	98
Degrees of Freedom	96
Centered R^2	0.7312754
R-Bar^2	0.7284761
Uncentered R^2	0.7312941
Mean of Dependent Variable	0.0109763389
Std Error of Dependent Variable	1.3210958382
Standard Error of Estimate	0.6883959762
Sum of Squared Residuals	45.493345926
Log Likelihood	-102.1373
Durbin-Watson Statistic	1.9183
Q(24-2)	12.3846
Significance Level of Q	0.9489196

Variable	Coeff	Std Error	T-Stat	Signif
1. AR{1}	1.062995267	0.097536434	10.89844	0.00000000
2. AR{2}	-0.265837108	0.097852464	-2.71671	0.00782164

3	0.46677	0.37008	0.81107	1.06300	-0.26584	204.27463

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in 5 Iterations. Final criterion was 0.0000072 <= 0.0000100

Dependent Variable ESERIES

Annual Data From 1875:01 To 1972:01

Usable Observations	98
Degrees of Freedom	96
Centered R^2	0.7316185

```
R-Bar^2                           0.7288228

Uncentered R^2                    0.7316371

Mean of Dependent Variable       0.0109727883

Std Error of Dependent Variable  1.3218885372

Standard Error of Estimate       0.6883691636

Sum of Squared Residuals         45.489802121

Log Likelihood                     -102.1346

Durbin-Watson Statistic               1.9176

Q(24-2)                              12.3184

Significance Level of Q           0.9504511


    Variable                      Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                       1.064396791  0.097489496     10.91807  0.00000000

2.  AR{2}                      -0.267421960  0.097804872     -2.73424  0.00744607



Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     5 Iterations. Final criterion was 0.0000072 <= 0.0000100

Dependent Variable ESERIES

Annual Data from 1875:01 To 1972:01

Usable Observations                     98

Degrees of Freedom                      96

Centered R^2                      0.7316185

R-Bar^2                           0.7288228

Uncentered R^2                    0.7316371

Mean of Dependent Variable       0.0109727883

Std Error of Dependent Variable 1.3218885372

Standard Error of Estimate       0.6883691636
```

```
Sum of Squared Residuals        45.489802121

Log Likelihood                     -102.1346

Durbin-Watson Statistic               1.9176

Q(24-2)                              12.3184

Significance Level of Q            0.9504511


    Variable                         Coeff      Std Error      T-Stat      Signif

************************************************************************************

1.  AR{1}                      1.064396791  0.097489496     10.91807  0.00000000

2.  AR{2}                     -0.267421960  0.097804872     -2.73424  0.00744607

4       0.46180       0.38470       0.81789       1.06440      -0.26742     204.26913
```

The BOXJENK can also directly estimate the model skipping over some set of observations. This is done either if the dependent variable has actual missing data in it, or if the SMPL option (a contraction of SaMPLe) is used to tag the observations which are to be included in the estimated sample. The results are slightly different because the EM algorithm used above is the modified version, which won't have quite the same stationary point as the full EM.

```
boxjenk(ar=2,demean,maxl,smpl=.not.miss) lake

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in     5 Iterations. Final criterion was 0.0000022 <= 0.0000100

Dependent Variable LAKE

Annual Data From 1875:01 To 1972:01

Usable Observations                     88

Degrees of Freedom                      86

Skipped/Missing (from 98)               10

Centered R^2                     0.7044048

R-Bar^2                          0.7009676

Uncentered R^2                   0.9937747

Mean of Dependent Variable    9.0411363636
```

```
Std Error of Dependent Variable 1.3337023327
Standard Error of Estimate       0.7293198032
Sum of Squared Residuals         45.744034277
Log Likelihood                      -100.2584
Durbin-Watson Statistic                1.9009
Q(24-2)                                8.7911
Significance Level of Q             0.9943525

    Variable                        Coeff      Std Error      T-Stat      Signif
*************************************************************************************
1.  AR{1}                        0.997774590  0.102097051      9.77281  0.00000000
2.  AR{2}                       -0.205989092  0.102110635     -2.01731  0.04678128
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
 Example 8.8.7 from pp 306-309

open data goals.tsm
data(format=free,org=columns) 1 57 england year

print
```

ENTRY	LAKE	MISS	CLAKE	ESERIES	ENGLAND	YEAR
1875:01	10.38	0	1.375918367347	1.375918367347	0	1872
1876:01	11.86	0	2.855918367347	2.855918367347	1	1874
1877:01	10.97	0	1.965918367347	1.965918367347	0	1876
1878:01	10.80	0	1.795918367347	1.795918367347	2	1878

1879:01	9.79	0	0.785918367347	0.785918367347	4	1880
1880:01	10.39	0	1.385918367347	1.385918367347	1	1882
1881:01	10.42	0	1.415918367347	1.415918367347	0	1884
1882:01	10.82	0	1.815918367347	1.815918367347	1	1886
1883:01	11.40	0	2.395918367347	2.395918367347	5	1888
1884:01	11.32	0	2.315918367347	2.315918367347	1	1890
1885:01	11.44	0	2.435918367347	2.435918367347	4	1892
1886:01	11.68	0	2.675918367347	2.675918367347	2	1894
1887:01	11.17	0	2.165918367347	2.165918367347	1	1896
1888:01	10.53	0	1.525918367347	1.525918367347	3	1898
1889:01	10.01	0	1.005918367347	1.005918367347	1	1900
1890:01	9.91	0	0.905918367347	0.905918367347	1	1904
1891:01	9.14	1	0.135918367347	0.461549385616	1	1906
1892:01	9.16	0	0.155918367347	0.155918367347	1	1908
1893:01	9.55	0	0.545918367347	0.545918367347	0	1910
1894:01	9.67	0	0.665918367347	0.665918367347	1	1912
1895:01	8.44	0	-0.564081632653	-0.564081632653	1	1914
1896:01	8.24	0	-0.764081632653	-0.764081632653	NA	1916
1897:01	9.10	0	0.095918367347	0.095918367347	NA	1918

1898:01	9.09	1	0.085918367347	0.385408201026	0	1921
1899:01	9.35	0	0.345918367347	0.345918367347	2	1923
1900:01	8.82	0	-0.184081632653	-0.184081632653	0	1925
1901:01	9.32	0	0.315918367347	0.315918367347	2	1927
1902:01	9.01	0	0.005918367347	0.005918367347	0	1929
1903:01	9.00	0	-0.004081632653	-0.004081632653	0	1931
1904:01	9.80	0	0.795918367347	0.795918367347	1	1933
1905:01	9.83	1	0.825918367347	0.818209259140	0	1935
1906:01	9.72	0	0.715918367347	0.715918367347	1	1937
1907:01	9.89	0	0.885918367347	0.885918367347	2	1939
1908:01	10.01	0	1.005918367347	1.005918367347	NA	1941
1909:01	9.37	0	0.365918367347	0.365918367347	NA	1943
1910:01	8.69	0	-0.314081632653	-0.314081632653	NA	1945
1911:01	8.19	0	-0.814081632653	-0.814081632653	2	1948
1912:01	8.67	1	-0.334081632653	-0.115804283199	1	1950
1913:01	9.55	0	0.545918367347	0.545918367347	2	1952
1914:01	8.92	0	-0.084081632653	-0.084081632653	4	1954
1915:01	8.09	0	-0.914081632653	-0.914081632653	1	1956
1916:01	9.37	0	0.365918367347	0.365918367347	4	1958

1917:01	10.13	0	1.125918367347	1.125918367347	1	1960
1918:01	10.14	0	1.135918367347	1.135918367347	0	1962
1919:01	9.51	1	0.505918367347	0.744665737979	0	1964
1920:01	9.24	0	0.235918367347	0.235918367347	4	1966
1921:01	8.66	0	-0.344081632653	-0.344081632653	1	1968
1922:01	8.86	0	-0.144081632653	-0.144081632653	0	1970
1923:01	8.05	0	-0.954081632653	-0.954081632653	1	1972
1924:01	7.79	0	-1.214081632653	-1.214081632653	0	1974
1925:01	6.75	0	-2.254081632653	-2.254081632653	1	1976
1926:01	6.75	1	-2.254081632653	-1.912574190480	1	1978
1927:01	7.82	0	-1.184081632653	-1.184081632653	2	1980
1928:01	8.64	0	-0.364081632653	-0.364081632653	1	1982
1929:01	10.58	0	1.575918367347	1.575918367347	1	1984
1930:01	9.48	0	0.475918367347	0.475918367347	0	1985
1931:01	7.38	0	-1.624081632653	-1.624081632653	0	1987
1932:01	6.90	0	-2.104081632653	-2.104081632653	NA	NA
1933:01	6.94	1	-2.064081632653	-2.519583386907	NA	NA
1934:01	6.24	0	-2.764081632653	-2.764081632653	NA	NA
1935:01	6.84	0	-2.164081632653	-2.164081632653	NA	NA

1936:01	6.85	0	-2.154081632653	-2.154081632653	NA	NA
1937:01	6.90	0	-2.104081632653	-2.104081632653	NA	NA
1938:01	7.79	0	-1.214081632653	-1.214081632653	NA	NA
1939:01	8.18	0	-0.824081632653	-0.824081632653	NA	NA
1940:01	7.51	1	-1.494081632653	-1.371839042102	NA	NA
1941:01	7.23	0	-1.774081632653	-1.774081632653	NA	NA
1942:01	8.42	0	-0.584081632653	-0.584081632653	NA	NA
1943:01	9.61	0	0.605918367347	0.605918367347	NA	NA
1944:01	9.05	0	0.045918367347	0.045918367347	NA	NA
1945:01	9.26	0	0.255918367347	0.255918367347	NA	NA
1946:01	9.22	0	0.215918367347	0.215918367347	NA	NA
1947:01	9.38	1	0.375918367347	0.287656841411	NA	NA
1948:01	9.10	0	0.095918367347	0.095918367347	NA	NA
1949:01	7.95	0	-1.054081632653	-1.054081632653	NA	NA
1950:01	8.12	0	-0.884081632653	-0.884081632653	NA	NA
1951:01	9.75	0	0.745918367347	0.745918367347	NA	NA
1952:01	10.85	0	1.845918367347	1.845918367347	NA	NA
1953:01	10.41	0	1.405918367347	1.405918367347	NA	NA
1954:01	9.96	1	0.955918367347	1.036846570883	NA	NA

1955:01	9.61	0	0.605918367347	0.605918367347	NA	NA
1956:01	8.76	0	-0.244081632653	-0.244081632653	NA	NA
1957:01	8.18	0	-0.824081632653	-0.824081632653	NA	NA
1958:01	7.21	0	-1.794081632653	-1.794081632653	NA	NA
1959:01	7.13	0	-1.874081632653	-1.874081632653	NA	NA
1960:01	9.10	0	0.095918367347	0.095918367347	NA	NA
1961:01	8.25	0	-0.754081632653	-0.754081632653	NA	NA
1962:01	7.91	0	-1.094081632653	-1.094081632653	NA	NA
1963:01	6.89	0	-2.114081632653	-2.114081632653	NA	NA
1964:01	5.96	0	-3.044081632653	-3.044081632653	NA	NA
1965:01	6.80	0	-2.204081632653	-2.204081632653	NA	NA
1966:01	7.68	0	-1.324081632653	-1.324081632653	NA	NA
1967:01	8.38	0	-0.624081632653	-0.624081632653	NA	NA
1968:01	8.52	0	-0.484081632653	-0.484081632653	NA	NA
1969:01	9.74	0	0.735918367347	0.735918367347	NA	NA
1970:01	9.31	0	0.305918367347	0.305918367347	NA	NA
1971:01	9.89	0	0.885918367347	0.885918367347	NA	NA
1972:01	9.96	0	0.955918367347	0.955918367347	NA	NA

```
sample(smpl=%valid(england)) england / scores

stats scores
```

```
Statistics on Series SCORES
Annual Data From 1875:01 To 1926:01
Observations                52
Sample Mean           1.269231      Variance              1.651584
Standard Error        1.285140      SE of Sample Mean     0.178217
t-Statistic (Mean=0)  7.121836      Signif Level (Mean=0) 0.000000
Skewness              1.258133      Signif Level (Sk=0)   0.000322
Kurtosis (excess)     1.036621      Signif Level (Ku=0)   0.154225
Jarque-Bera          16.046724      Signif Level (JB=0)   0.000328
```

A "static" Poisson model can be estimated by using DDV (Discrete Dependent Variables) with TYPE=COUNT. With a fixed probability, the only explanatory variable is the constant. Note that this shows the over dispersion by the standardized variance, which should be 1 if a Poisson is actually correct.

```
ddv(type=count) scores

# constant
```

```
Poisson Regression - Estimation by Newton-Raphson
Convergence in     4 Iterations. Final criterion was  0.0000001 <=  0.0000100
Dependent Variable SCORES
Annual Data From 1875:01 To 1926:01
Usable Observations                        52
Degrees of Freedom                         51
R^2                                         2
Standardized Variance               1.2762238
Log Likelihood                       -50.2649
```

```
   Variable                        Coeff      Std Error      T-Stat      Signif
*******************************************************************************
1.  Constant                    0.2384110234 0.1230914842      1.93686  0.05276241
```

```
alpha and lambda will be generated recursively during evaluations of
* the likelihood function.

function %LogBetaPascal y a b n
type real %LogBetaPascal y a b n
compute %LogBetaPascal=-log(y+n)+%lnbeta(n+a,y+b)-%lnbeta(n,y+1)-%lnbeta(a,b)

set alpha  = scores
set lambda = 0.0

We have to give an initial guess value for delta because at the
* default (zero), the likelihood function isn't defined.

compute delta=.500
```

This is the main formula for updating the two "state" variables, and evaluating the log likelihood. Note that there's a typo in the book: the final parameter in the nb density should be lambda/(1+lambda).

```
This sets the pre-sample values for lambda and delta. A "startup"
* formula like this is executed just once per function evaluation. This
* is the sequence which generates the estimates in the data. You
* generated this by starting at lambda and alpha=0, then updating for
* the first two data points (with 0 and 1 goal respectively), with the
```

```
* estimation starting at period 3.

frml init = lambda{1}=delta*(1+delta),alpha{1}=delta*scores{1}

The likelihood function is maximized over the range 3-52. You can't
 start earlier than 3, because 2 is the first period with a non-zero value for
"score".
maximize(startup=init,method=bfgs) logl 3 52
Compute the expected scores over the sample period

set escore = alpha/lambda
graph(footer="Figure 8-10 One-step predictors of goal data",overlay=dots,ovsame) 2
# escore
# scores

This will compute the predicted probabilities given the parameters at
* the end of the period.
*
dec vect predicted(6)
ewise predicted(i)=%negbin(i-1,alpha(52),lambda(52)/(1+lambda(52)))
disp predicted

Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 9.2.1 from page 324
open data deaths.tsm
calendar(m) 1973
data(format=free,org=columns) 1973:1 1978:12 deaths
The Holt-Winters (non-seasonal) model is done with the ESMOOTH
* instruction with the option TREND=LINEAR. (TREND=NONE will do a simple
* "local level" exponential smoothing). The other options here request
```

```
*   (a) estimation with the parameters restricted to the range [0,1]
*      (the default will allow them to stray outside of that, as the
*       recursions are stable for, for instance, alpha values up to 2.0).
*   (b) initialization of the recursions based upon the just the values
*       at the start of the data set (the default uses the full data set).


* The FORECASTS and STEPS options put 24 steps of out-of-sample
* forecasts into the series FORECAST.

esmooth(trend=linear,constrain,init=start,forecasts=forecast,steps=24) deaths

Exponential Smoothing for Series DEATHS
Model with TREND=Linear , SEASONAL=None
Alpha (level) 1.000000
Gamma (trend) 0.109470



graph(footer="Figure 9-2 Deaths data with predictions of non-seasonal HW") 2
# deaths
# forecast
```

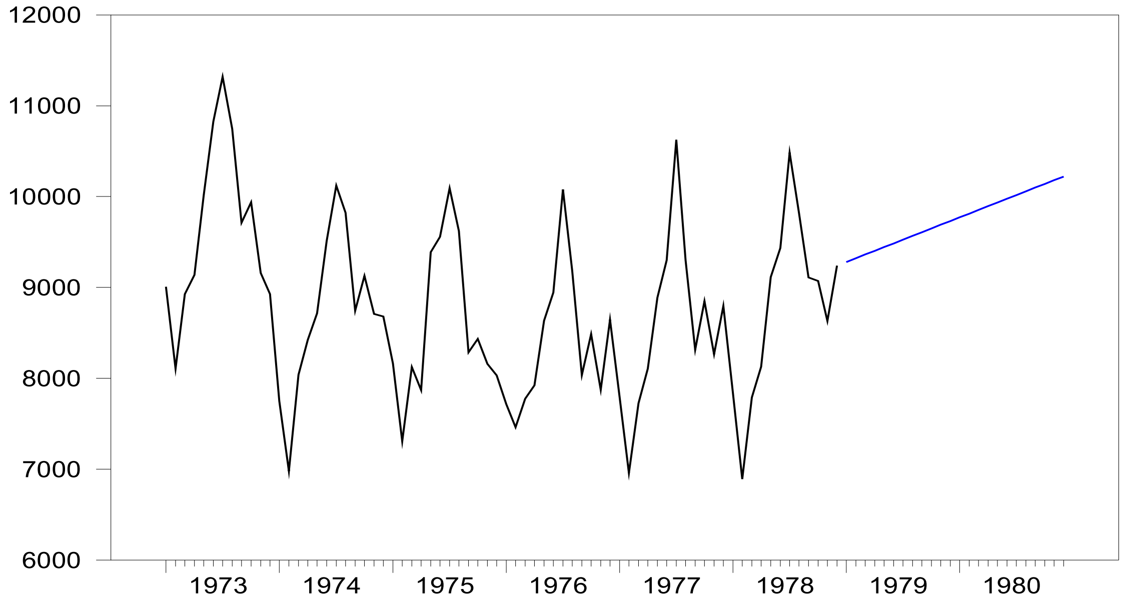


Figure 9-2 Deaths data with predictions of non-seasonal HW

```
data(unit=input) 1979:1 1979:6 adeaths

 7798 7406 8363 8460 9217 9316

Compute statistics on forecast errors and display the forecasts and

* actual data.


@uforeerrors(title="Holt-Winters Forecasts") adeaths forecast 1979:1 1979:6

print(picture="*.") 1979:1 1979:6 adeaths forecast


Holt-Winters Forecasts

From 1979:01 to 1979:06

Mean Error                          -956.4617

Mean Absolute Error                  956.4617

Root Mean Square Error              1143.0471

Mean Square Error          1306556.654380


Mean Pct Error                      -0.120462
```

```
Mean Abs Pct Error               0.120462

Root Mean Square Pct Error       0.147618


 ENTRY        ADEATHS        FORECAST

 1979:01         7798            9281

 1979:02         7406            9322

 1979:03         8363            9363

 1979:04         8460            9404

 1979:05         9217            9444

 1979:06         9316            9485
```

```
Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 9.2.1 from page 324
open data deaths.tsm
calendar(m) 1973
data(format=free,org=columns) 1973:1 1978:12 deaths
he Holt-Winters (non-seasonal) model is done with the ESMOOTH
* instruction with the option TREND=LINEAR. (TREND=NONE will do a simple
* "local level" exponential smoothing). The other options here request
*   (a) estimation with the parameters restricted to the range [0,1]
*      (the default will allow them to stray outside of that, as the
*       recursions are stable for, for instance, alpha values up to 2.0).
*   (b) initialization of the recursions based upon the just the values
*       at the start of the data set (the default uses the full data set).
```

```
esmooth (trend=linear,constrain,init=start,forecasts=forecast,steps=24) deaths

Exponential Smoothing for Series DEATHS

Model with TREND=Linear, SEASONAL=None

Alpha (level) 1.000000

Gamma (trend) 0.109470
```

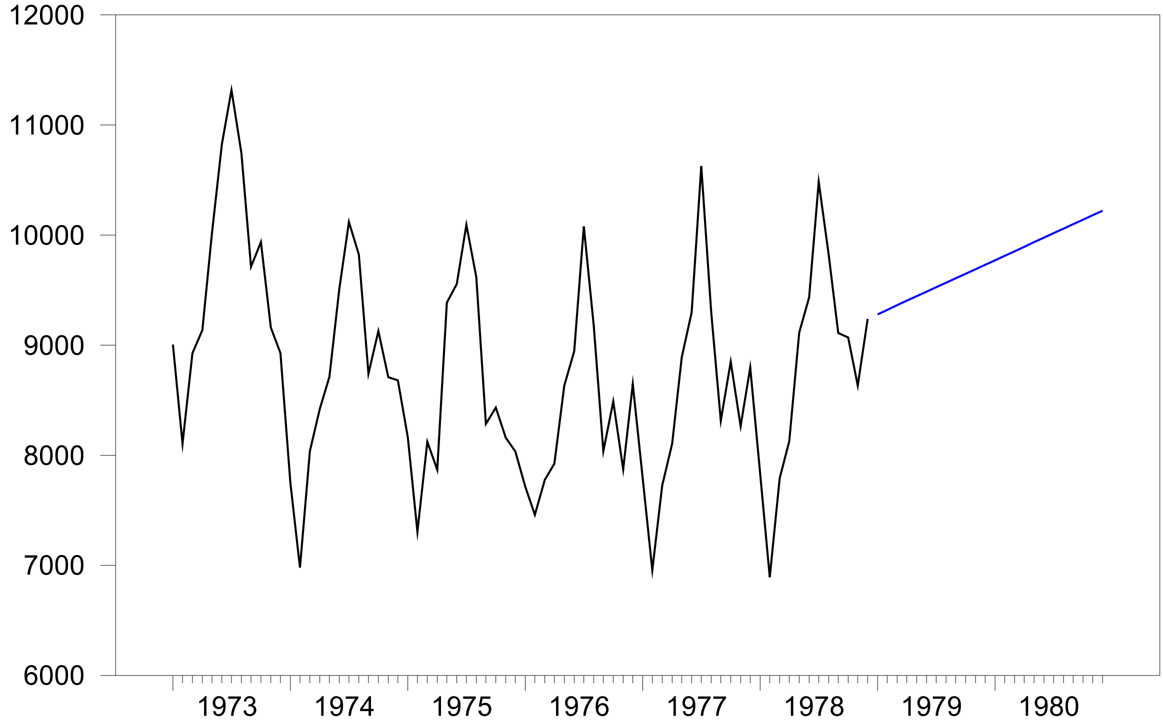
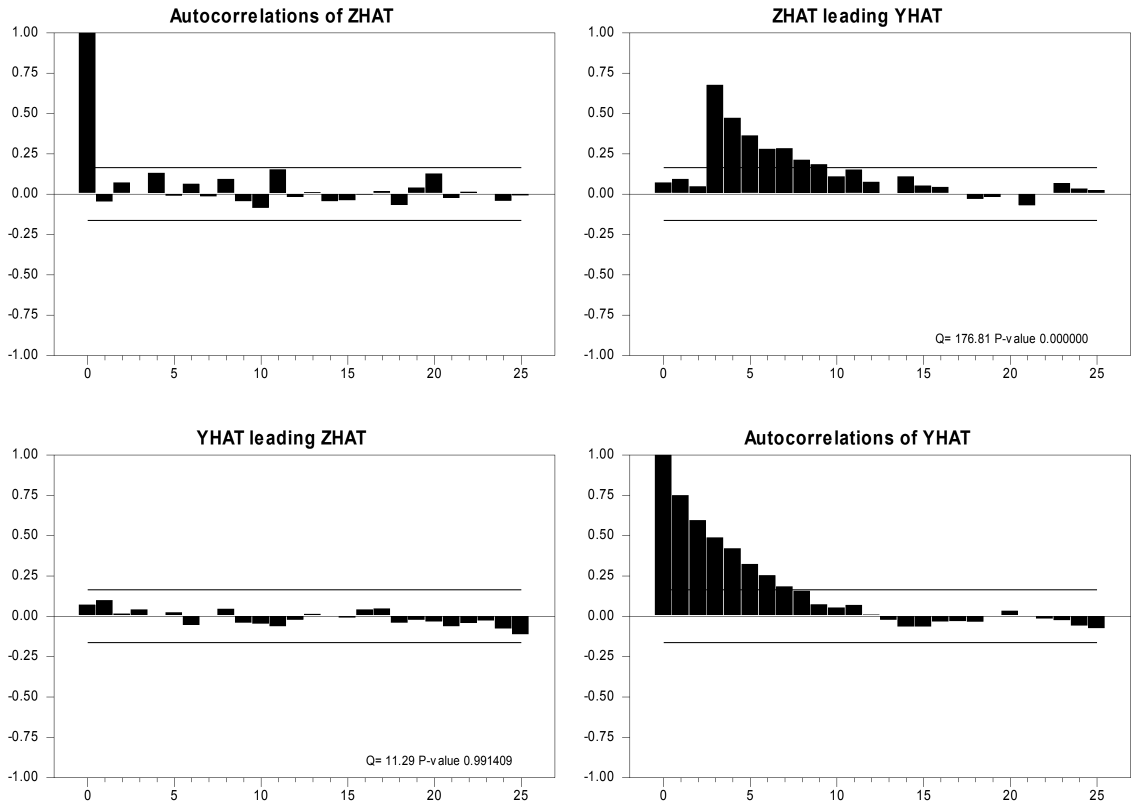


Figure 9-2 Deaths data with predictions of non-seasonal HW

```
Compute statistics on forecast errors and display the forecasts and

* actual data.

*

* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 10.1.1 from pp 335-338
```

```
open data ls2.tsm

data(format=free,org=columns) 1 150 lead sales

Difference and pull the means out of the two variables

diff(diffs=1,center) lead / x1

diff(diffs=1,center) sales / x2

Fix an MA(1) model to the transformed "lead" series

boxjenk(ma=1,maxl) x1 / zhat

boxjenk(ma=1,maxl) x1 / zhat

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in    10 Iterations. Final criterion was  0.0000075 <=  0.0000100

Dependent Variable X1

Usable Observations                      149

Degrees of Freedom                       148

Centered R^2                       0.2153471

R-Bar^2                            0.2153471

Uncentered R^2                     0.2153471

Mean of Dependent Variable      0.0000000000

Std Error of Dependent Variable  0.3162253282

Standard Error of Estimate      0.2801144044

Sum of Squared Residuals        11.612683771

Log Likelihood                      -21.4366

Durbin-Watson Statistic               2.0999

Q(36-1)                              46.7195

Significance Level of Q            0.0889844


    Variable                        Coeff      Std Error      T-Stat      Signif

*************************************************************************************

1.  MA{1}                       -0.474354402  0.072581465     -6.53548  0.00000000
```

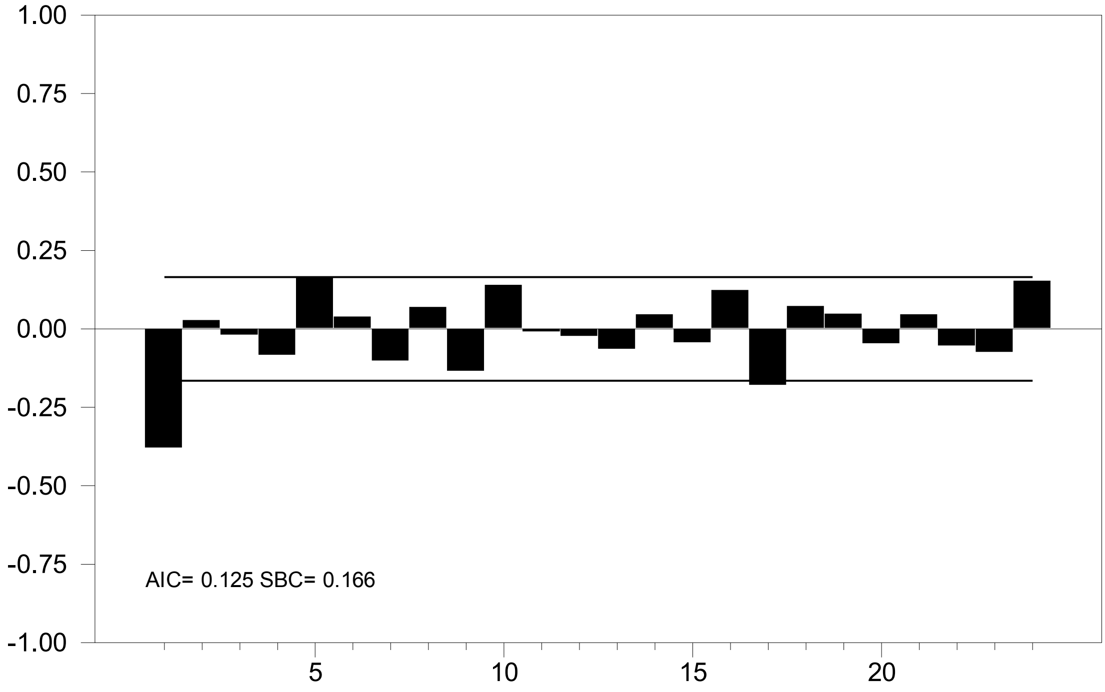
```
* This use of BOXJENK will filter the series x2 by the same MA(1) filter
* estimated for x1. METHOD=EVAL just does a single evaluation at the
* initial values provided in the %beta (estimated coefficients) vector.
* The residuals are the result of the filter.
boxjenk(ma=1,maxl,initial=%beta,method=eval) x2 / yhat

@crosscorr zhat yhat
```




```
* This does the direct estimate of the coefficients in the transfer
* function.
```

```
boxjenk(inputs=1,maxl) x2
# x1 0 1 3
@regcorrs
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     7 Iterations. Final criterion was  0.0000001 <=  0.0000100
Dependent Variable X2
Usable Observations                        146
Degrees of Freedom                         144
Centered R^2                         0.9692466
R-Bar^2                              0.9690330
Uncentered R^2                       0.9692507
Mean of Dependent Variable        0.0168520732
Std Error of Dependent Variable 1.4536524130
Standard Error of Estimate        0.2558056438
Sum of Squared Residuals          9.4228599476
Log Likelihood                         -7.1109
Durbin-Watson Statistic                 2.7383
Q(36-0)                                77.5504
Significance Level of Q              0.0000711

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  N_X1{3}                      4.6801461591 0.0755782190     61.92454  0.00000000
2.  D_X1{1}                      0.7264140921 0.0074027790     98.12722  0.00000000
```




```
boxjenk(inputs=1,ma=1,maxl) x2
# x1 0 1 3
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in      9 Iterations. Final criterion was  0.0000099 <=  0.0000100
Dependent Variable X2
Usable Observations                        146
Degrees of Freedom                         143
Centered R^2                         0.9748041
R-Bar^2                              0.9744517
Uncentered R^2                       0.9748075
Mean of Dependent Variable        0.0168520732
Std Error of Dependent Variable   1.4536524130
```

```
Standard Error of Estimate       0.2323492064
Sum of Squared Residuals         7.7200199804
Log Likelihood                         7.3076
Durbin-Watson Statistic                1.9412
Q(36-1)                               33.4455
Significance Level of Q             0.5432172

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  MA{1}                        -0.481664455  0.074195352     -6.49184  0.00000000
2.  N_X1{3}                       4.700107487  0.059631823     78.81878  0.00000000
3.  D_X1{1}                       0.725487097  0.004551488    159.39558  0.00000000

* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 10.1.2 from pp 338-339
open data ls2.tsm
data(format=free,org=columns) 1 150 lead sales
diff(diffs=1,center) lead / x1
compute leadmean=%mean
diff(diffs=1,center) sales / x2
compute salesmean=%mean

boxjenk(ma=1,maxl,define=leadeq) x1 / wlead
boxjenk(inputs=1,ma=1,maxl,define=saleseq) x2 / wsales
# x1 0 1 3


Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in    10 Iterations. Final criterion was  0.0000075 <=  0.0000100
```

Dependent Variable X1

Usable Observations	149
Degrees of Freedom	148
Centered R^2	0.2153471
R-Bar^2	0.2153471
Uncentered R^2	0.2153471
Mean of Dependent Variable	0.0000000000
Std Error of Dependent Variable	0.3162253282
Standard Error of Estimate	0.2801144044
Sum of Squared Residuals	11.612683771
Log Likelihood	-21.4366
Durbin-Watson Statistic	2.0999
Q(36-1)	46.7195
Significance Level of Q	0.0889844

Variable	Coeff	Std Error	T-Stat	Signif
1. MA{1}	-0.474354402	0.072581465	-6.53548	0.00000000

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in 9 Iterations. Final criterion was 0.0000099 <= 0.0000100

Dependent Variable X2

Usable Observations	146
Degrees of Freedom	143
Centered R^2	0.9748041
R-Bar^2	0.9744517
Uncentered R^2	0.9748075
Mean of Dependent Variable	0.0168520732

```
Std Error of Dependent Variable  1.4536524130
Standard Error of Estimate       0.2323492064
Sum of Squared Residuals         7.7200199804
Log Likelihood                         7.3076
Durbin-Watson Statistic                1.9412
Q(36-1)                               33.4455
Significance Level of Q             0.5432172

   Variable                        Coeff      Std Error      T-Stat      Signif
*************************************************************************************
1.  MA{1}                     -0.481664455  0.074195352     -6.49184  0.00000000
2.  N_X1{3}                    4.700107487  0.059631823     78.81878  0.00000000
3.  D_X1{1}                    0.725487097  0.004551488    159.39558  0.00000000

* Generate identities to substitute back from the differenced series to
* levels. (This does automatically what is being done manually at the
* top of page 339).

equation(identity,coeffs=||1.0,1.0,leadmean||) leadid lead
# lead{1} x1 constant
equation(identity,coeffs=||1.0,1.0,salesmean||) salesid sales
# sales{1} x2 constant

equation(identity,coeffs=||1.0,1.0,leadmean||) leadid lead
# lead{1} x1 constant
equation(identity,coeffs=||1.0,1.0,salesmean||) salesid sales
# sales{1} x2 constant
* Group the two estimated equations and the undifferencing identities
* into a model.
```

```
forecast(model=tmodel,steps=10,results=forecast,stderrs=stderrs)

print / forecast
print / stderrs
```

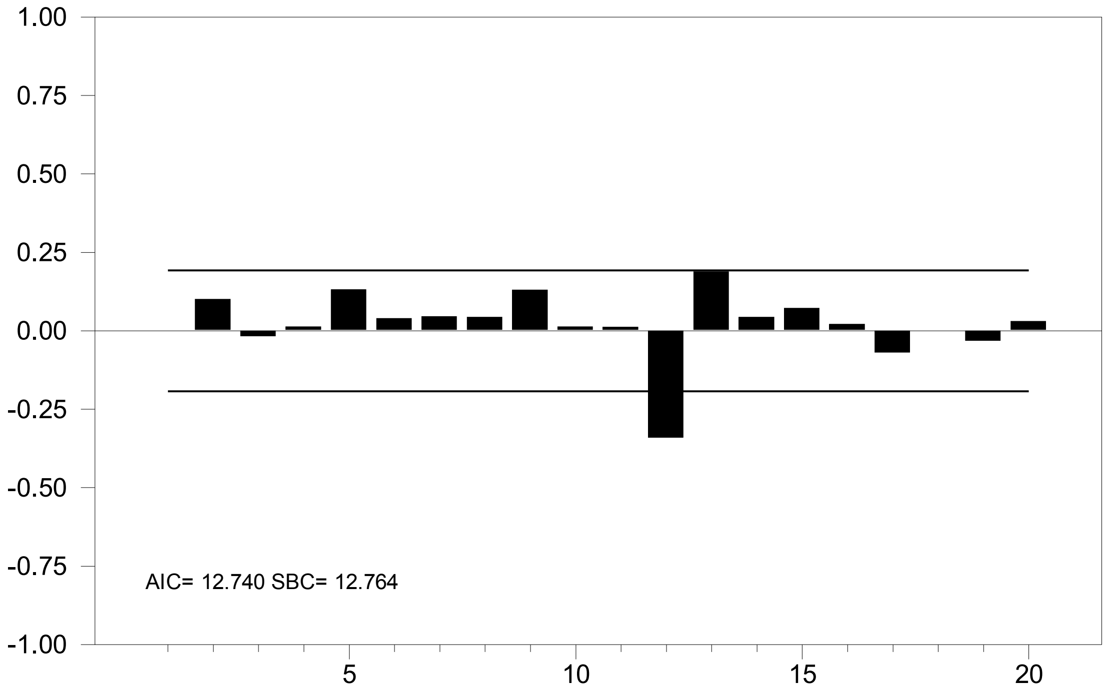
ENTRY	FORECAST(1)	FORECAST(2)	FORECAST(3)	FORECAST(4)
151	0.137866351488	-0.219778988625	13.56061802934	262.900355240
152	0.000000000000	0.921383102482	13.58336970719	264.241872570
153	0.000000000000	-1.177523549862	13.60612138505	263.484483249
154	0.000000000000	-0.206291470441	13.62887306290	263.698326006
155	0.000000000000	-0.149661799933	13.65162474075	263.968798435
156	0.000000000000	-0.108577704697	13.67437641860	264.280354958
157	0.000000000000	-0.078771723731	13.69712809645	264.621717463
158	0.000000000000	-0.057147869139	13.71987977431	264.984703822
159	0.000000000000	-0.041460041656	13.74263145216	265.363378008
160	0.000000000000	-0.030078725244	13.76538313001	265.753433511

ENTRY	STDERRS(1)	STDERRS(2)	STDERRS(3)	STDERRS(4)
151	0.280114404372	0.232349206388	0.280114404372	0.232349206388
152	0.310031364176	0.257897220399	0.316455309954	0.261707293429
153	0.310031364176	0.257897220399	0.349032730356	0.288088981321
154	0.310031364176	1.341589271232	0.378818862415	1.353089284297
155	0.310031364176	1.381730834687	0.406427870824	2.135094002421
156	0.310031364176	1.402397155391	0.432277107703	2.852046377214
157	0.310031364176	1.413153097669	0.456665502864	3.520906270624
158	0.310031364176	1.418781535926	0.479815865898	4.146777540475

```
    159   0.310031364176  1.421735010203  0.501899540575  4.733268437790

    160   0.310031364176  1.423287055036  0.523051653750  5.283860593212

* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 10.2.1 from pp 341-342

open data sbl2.tsm
calendar(m) 1975
data(format=free,org=columns) 1975:1 1984:12 step deaths

The step function could have been created with
* set step = t>=1983:2
* You can do this with a LINREG instruction, but it will be more
* convenient here to just use BOXJENK, since it handles bookkeeping for
* the seasonal differencing.
The step function could have been created with
* set step = t>=1983:2
boxjenk(sdiff=1,apply,reg) deaths / resids
# step
@regcorrs
Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in     2 Iterations. Final criterion was  0.0000000 <=  0.0000100
Dependent Variable DEATHS
Monthly Data From 1976:01 To 1984:12
Usable Observations                       108
Degrees of Freedom                        107
Centered R^2                        0.6968544
R-Bar^2                             0.6968544
Uncentered R^2                      0.9921519
```

Mean of Dependent Variable	1559.6018519
Std Error of Dependent Variable	255.4379761
Standard Error of Estimate	140.6407207
Sum of Squared Residuals	2116439.9167
Log Likelihood	-686.9336
Durbin-Watson Statistic	1.9509
Q(27-0)	37.3896
Significance Level of Q	0.0880217

Variable	Coeff	Std Error	T-Stat	Signif
1. STEP	-346.9166671	40.5994791	-8.54486	0.00000000



Find the best fitting MA model for the residuals

```
@bjautofit(qmax=13,pmax=0) resids
```

```
AIC analysis of models for series RESIDS

   MA

AR    0         1         2         3         4         5         6         7
8         9        10        11        12         13

 0 1373.8671 1375.8624 1376.5586 1378.2936 1380.2783 1380.7006 1380.9814 1380.2352
1379.5212 1380.7342 1381.4748 1382.8060 1363.3945* 1365.0394
```

```
* This fits the MA(12) noise term with the intervention variable. This
* is a very difficult model to fit and the BOXJENK instruction is unable
* to estimate it by maximum likelihood, so we take the MAXL option out.
```

```
boxjenk(sdiff=1,ma=12,reg,applydifferences) deaths
# step
```

```
Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in    15 Iterations. Final criterion was  0.0000096 <=  0.0000100
Dependent Variable DEATHS
Monthly Data From 1976:01 To 1984:12
Usable Observations                        108
Degrees of Freedom                          95
Centered R^2                         0.7927802
R-Bar^2                              0.7666051
Uncentered R^2                       0.9946353
```

```
Mean of Dependent Variable       1559.6018519
Std Error of Dependent Variable   255.4379761
Standard Error of Estimate        123.4045485
Sum of Squared Residuals         1446724.8452
Log Likelihood                      -666.3902
Durbin-Watson Statistic                2.0164
Q(27-12)                               9.5676
Significance Level of Q             0.8460205

    Variable                        Coeff      Std Error      T-Stat      Signif
***********************************************************************************
1.  MA{1}                         0.1632653    0.0828455      1.97072  0.05166647
2.  MA{2}                         0.0803746    0.0832459      0.96551  0.33674172
3.  MA{3}                         0.0420937    0.0841818      0.50003  0.61820783
4.  MA{4}                         0.0523845    0.0844344      0.62042  0.53646906
5.  MA{5}                         0.0525410    0.0847690      0.61981  0.53686392
6.  MA{6}                         0.1359907    0.0851107      1.59781  0.11340624
7.  MA{7}                         0.0101993    0.0855939      0.11916  0.90540064
8.  MA{8}                         0.0965127    0.0867377      1.11270  0.26864651
9.  MA{9}                         0.1042886    0.0877079      1.18904  0.23738559
10. MA{10}                       -0.0719249    0.0878311     -0.81890  0.41489293
11. MA{11}                        0.1515563    0.0881097      1.72009  0.08867276
12. MA{12}                       -0.6292036    0.0884333     -7.11501  0.00000000
13. STEP                       -317.9075539   52.6157233     -6.04206  0.00000003

This does an alternative noise term which just includes the seasonal
* MA, omitting MA(1),...,MA(11). This seems to provide an adequate model.
```

```
boxjenk(sdiff=1,sma=1,maxl,reg,applydifferences) deaths
# step

Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in    10 Iterations. Final criterion was  0.0000064 <=  0.0000100
Dependent Variable DEATHS
Monthly Data From 1976:01 To 1984:12
Usable Observations                        108
Degrees of Freedom                         106
Centered R^2                         0.7834652
R-Bar^2                              0.7814224
Uncentered R^2                       0.9943942
Mean of Dependent Variable        1559.6018519
Std Error of Dependent Variable    255.4379761
Standard Error of Estimate         119.4230874
Sum of Squared Residuals          1511758.6240
Log Likelihood                       -672.5986
Durbin-Watson Statistic                 1.7552
Q(27-1)                                18.9457
Significance Level of Q              0.8387131

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  SMA {12}                     -0.6873655    0.0904793     -7.59694 0.00000000
2.  STEP                       -298.1499366   28.0305460    -10.63661 0.00000000

* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 10.3.1 from pp 351-352
all 1000
```

```
compute a0=1.0,a1=0.5

* This draws the first value from the unconditional distribution (which
* is mean 0, variance a0/(1-a1)); the remaining ones are generated
* recursively using the previous value. Because this uses random
* numbers, you'll get somewhat different results each time you run this
* (and will also get somewhat different results from the ones in the
* book).
graph(footer="Figure 10-7 A Realization of an ARCH process")
# arch
```

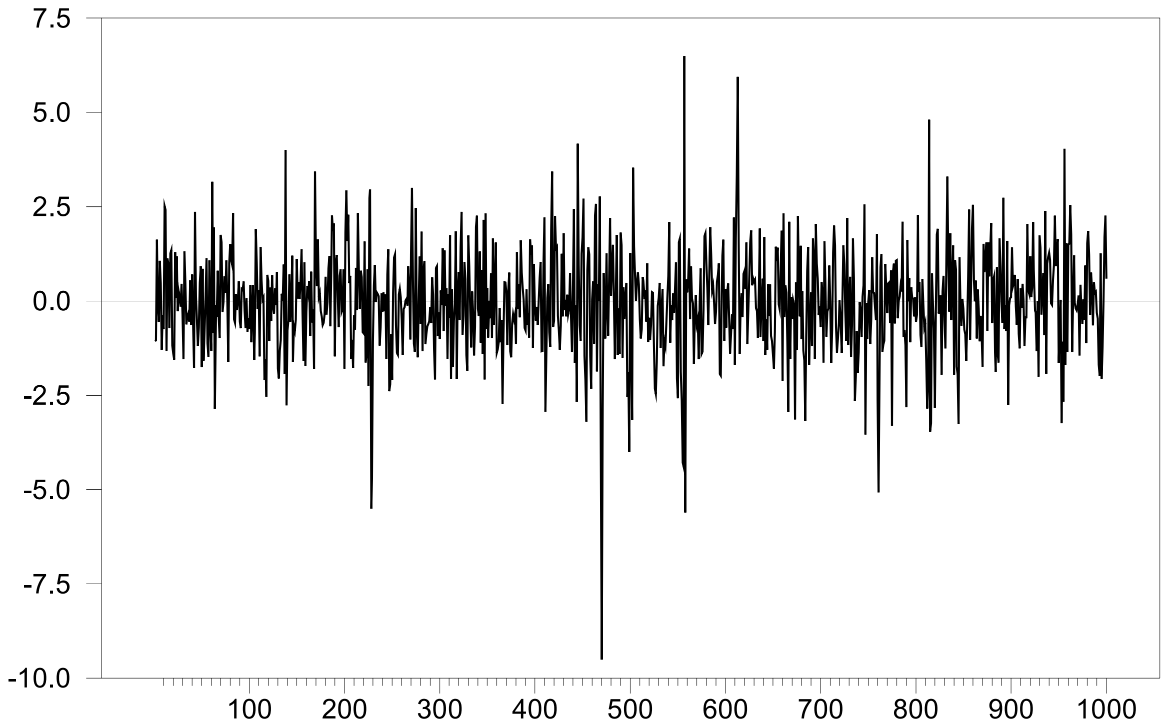
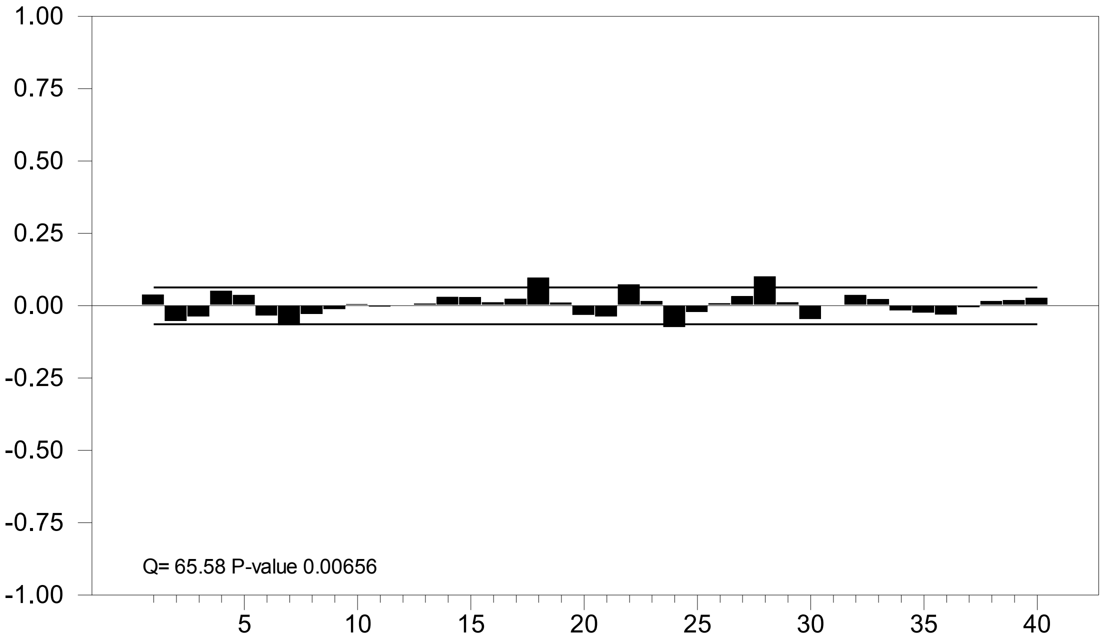


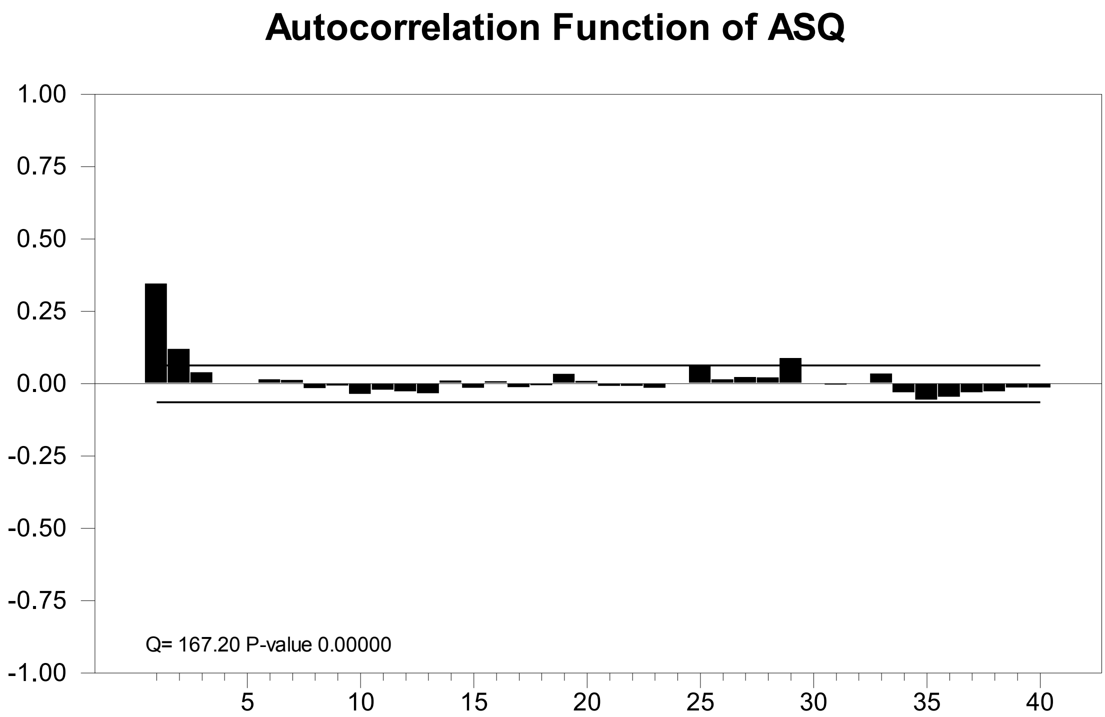
Figure 10-7 A Realization of an ARCH process

```
@acf(number=40) arch
set asq = arch^2
@acf(number=40) asq
```

Autocorrelation Function of ARCH




```
* Do independence tests
*
@bdindtests arch
@bdindtests arch


Independence Tests for Series ARCH
Test            Statistic  P-Value
Ljung-Box Q(63)   99.50363     0.0023
McLeod-Li(63)    232.30260     0.0000
Turning Points    -1.82666     0.0678
Difference Sign    0.93066     0.3520
Rank Test          0.71363     0.4755
```




```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 10.3.1 from pp 353-356

* The GARCH instruction, by default, will estimate the mean of the
* process along with the variance parameters. This will give a different
* estimate of the mean because it "downweights" the observations with a
* high variance.
garch(p=1,q=1,hseries=h) / dowj


GARCH Model - Estimation by BFGS
Convergence in    20 Iterations. Final criterion was  0.0000011 <=  0.0000100
```

```
Dependent Variable DOWJ
Usable Observations                         464
Log Likelihood                        -729.3299

    Variable                           Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  Mean                        0.1289278723 0.0487508204      2.64463  0.00817803
2.  C                           0.1207555416 0.0539664297      2.23760  0.02524684
3.  A                           0.1358078650 0.0401034973      3.38643  0.00070807
4.  B                           0.7904619916 0.0539835312     14.64265  0.00000000
Do the stacked graph of the returns with the estimated variances
*
spgraph(vfields=2,footer="Figure 10.10 Returns and GARCH Estimates of Variance")
graph
# dowj
graph
# h
spgraph(done)
* If the data are already mean zero (by, for instance, doing a
* DIFF(CENTER) instruction), you add the NOMEAN option to the GARCH
* instruction.
```

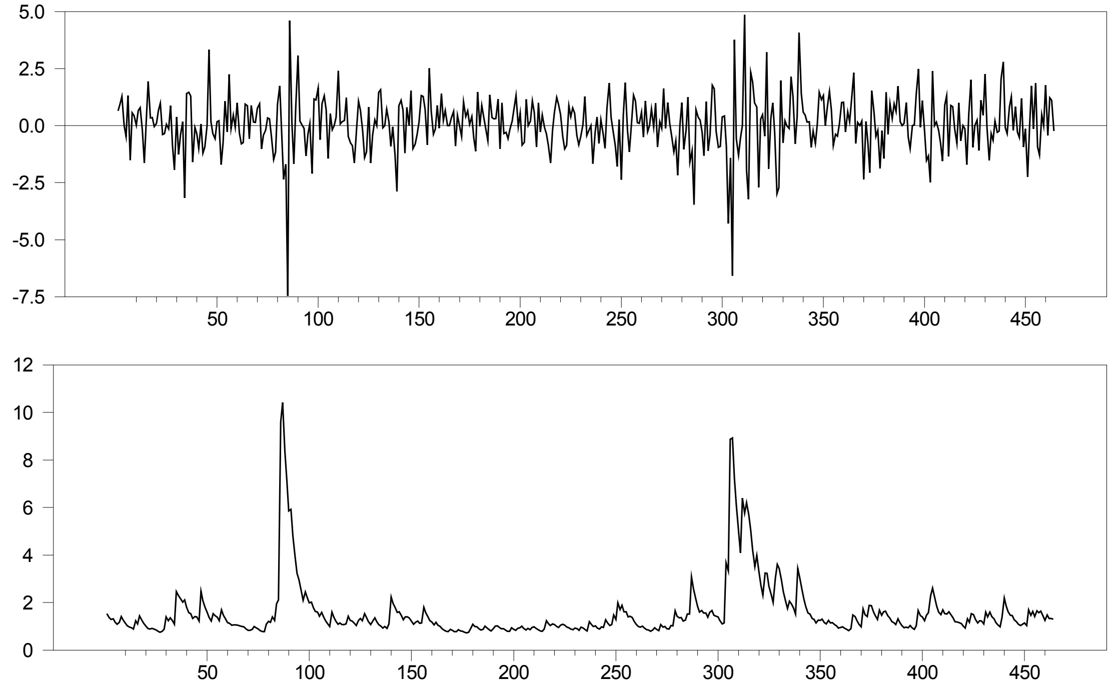


Figure 10.10 Returns and GARCH Estimates of Variance

```
diff(center) dowj / cdow

garch(p=1,q=1,nomean) / cdow

GARCH Model - Estimation by BFGS

Convergence in    19 Iterations. Final criterion was  0.0000006 <=  0.0000100

Dependent Variable CDOW

Usable Observations                        464

Log Likelihood                        -730.2009


   Variable                        Coeff       Std Error      T-Stat      Signif
*******************************************************************************
1.  C                           0.1200573886 0.0573421667      2.09370  0.03628657
2.  A                           0.1261749248 0.0352952634      3.57484  0.00035044
3.  B                           0.7992279701 0.0552615704     14.46264  0.00000000
```

```
* This estimates the GARCH model with a t distribution
garch(p=1,q=1,distrib=t,resids=u,hseries=h) / dowj

GARCH Model - Estimation by BFGS
Convergence in    26 Iterations. Final criterion was  0.0000039 <=  0.0000100
Dependent Variable DOWJ
Usable Observations                        464
Log Likelihood                       -713.0078

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  Mean                        0.1035064433 0.0470404127      2.20037  0.02778046
2.  C                           0.1091690109 0.0653768713      1.66984  0.09495073
3.  A                           0.0667618282 0.0298062459      2.23986  0.02509999
4.  B                           0.8566081896 0.0619819435     13.82029  0.00000000
5.  Shape                       5.5016685986 1.3552223946      4.05961  0.00004916

* Compute the standardized residuals
set stdu = u/sqrt(h)

* Test them for independence
*
@bdindtests(number=20) stdu
@bdindtests(number=20) stdu

Independence Tests for Series STDU
Test             Statistic  P-Value
Ljung-Box Q(20)  21.510805     0.3676
McLeod-Li(20)    15.756413     0.7316
```

```
Turning Points   -1.434153     0.1515

Difference Sign  -0.562254     0.5739

Rank Test         0.200181     0.8413


* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.

* Example 10.3.3 from page 356


open data sunspots.dat

calendar 1770

data(format=free,org=columns) 1770:1 1869:1 sunspots

*

@bjautofit(pmax=5,qmax=5,demean) sunspots
```

```
AIC analysis of models for series SUNSPOTS

   MA

AR    0        1        2        3        4         5

 0 1006.9298 902.9883 848.8760 839.5197 833.1630  834.6761

 1 1008.9298 844.9177 835.0123 835.8365 834.9126  834.0139

 2  833.2997 829.1183 830.7422 832.7398 896.7020  832.8997

 3  831.0521 830.8460 832.1951 834.5550 828.0790* 834.7358

 4  830.9797 833.1804 834.6328 836.5497 833.6939  830.7500

 5  832.0734 828.9307 839.6961 829.4047 830.3997  829.7774
```

```
* The ARMA(3,4) model has some local likelihood maxima, one of which

* BOXJENK finds rather than the (slightly) better set of coefficients

* shown in the text. We can get that one by starting out with

* conditional least squares estimation, then switching to ML.

boxjenk(ar=3,ma=4,demean) sunspots

boxjenk(ar=3,ma=4,demean,maxl,initial=%beta) sunspots
```

```
Box-Jenkins - Estimation by LS Gauss-Newton
Convergence in    34 Iterations. Final criterion was  0.0000027 <=  0.0000100
Dependent Variable SUNSPOTS
Annual Data From 1773:01 To 1869:01
Usable Observations                        97
Degrees of Freedom                         90
Centered R^2                        0.8592237
R-Bar^2                             0.8498386
Uncentered R^2                      0.9442286
Mean of Dependent Variable       45.814432990
Std Error of Dependent Variable  37.302348534
Standard Error of Estimate       14.454908062
Sum of Squared Residuals         18804.993038
Log Likelihood                      -393.0946
Durbin-Watson Statistic                1.9832
Q(24-7)                               10.4089
Significance Level of Q             0.8855946

    Variable                        Coeff      Std Error      T-Stat      Signif
*********************************************************************************
1.  AR{1}                        2.451617032  0.087378515     28.05744  0.00000000
2.  AR{2}                       -2.221683372  0.152320332    -14.58560  0.00000000
3.  AR{3}                        0.731313154  0.082538359      8.86028  0.00000000
4.  MA{1}                       -0.934583447  0.128909835     -7.24990  0.00000000
5.  MA{2}                       -0.335229111  0.154282015     -2.17283  0.03241986
6.  MA{3}                        0.324324700  0.154501240      2.09917  0.03860177
7.  MA{4}                        0.184997993  0.126677230      1.46039  0.14766546
```

```
Box-Jenkins - Estimation by ML Gauss-Newton
Convergence in     9 Iterations. Final criterion was  0.0000041 <=  0.0000100
Dependent Variable SUNSPOTS
Annual Data From 1770:01 To 1869:01
Usable Observations                       100
Degrees of Freedom                         93
Centered R^2                        0.8602621
R-Bar^2                             0.8512467
Uncentered R^2                      0.9461186
Mean of Dependent Variable       46.930000000
Std Error of Dependent Variable  37.365044703
Standard Error of Estimate       14.411153800
Sum of Squared Residuals         19314.365908
Log Likelihood                      -406.8271
Durbin-Watson Statistic                2.0156
Q(25-7)                               11.4266
Significance Level of Q             0.8753833

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  AR{1}                         2.464242546  0.086543241     28.47412  0.00000000
2.  AR{2}                        -2.249908223  0.151538024    -14.84715  0.00000000
3.  AR{3}                         0.756783911  0.082856841      9.13363  0.00000000
4.  MA{1}                        -0.949146259  0.132268868     -7.17589  0.00000000
5.  MA{2}                        -0.297213006  0.154855334     -1.91929  0.05801291
6.  MA{3}                         0.312477128  0.154071245      2.02813  0.04540707
7.  MA{4}                         0.139207196  0.129645496      1.07375  0.28571170
```

```
@bdindtests(number=20) %resids

Independence Tests for Series %RESIDS
Test            Statistic  P-Value
Ljung-Box Q(20) 13.6794256     0.8464
McLeod-Li(20)   52.2722695     0.0001
Turning Points   2.0743654     0.0380
Difference Sign  2.2404915     0.0251
Rank Test        0.3037679     0.7613
set resids = %resids

* Fit GARCH model to the residuals. The default for GARCH includes an
* estimated intercept, which we want to omit here since we're working
* with residuals which should be close to mean zero.
garch(p=1,q=1,nomean) / resids

GARCH Model - Estimation by BFGS
Convergence in    17 Iterations. Final criterion was  0.0000000 <=  0.0000100
Dependent Variable RESIDS
Annual Data From 1770:01 To 1869:01
Usable Observations                      100
Log Likelihood                     -398.7794

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  C                            16.123023929 21.643220627      0.74495  0.45630455
2.  A                             0.252221138  0.133918024      1.88340  0.05964625
3.  B                             0.679220858  0.187846711      3.61583  0.00029939
```

```
* Brockwell & Davis, Introduction to Time Series and Forecasting, 2nd ed.
* Example 10.5.1 on pp 363-365
* (Note-this takes several minutes to run)
open data nile.tsm
calendar 622
data(format=free,org=columns) 622:01 871:01 nile

print
```

```
 ENTRY         NILE
  622:01        1157
  623:01        1088
  624:01        1169
  625:01        1169
  626:01         984
  627:01        1322
  628:01        1178
  629:01        1103
  630:01        1211
  631:01        1292
  632:01        1124
  633:01        1171
  634:01        1133
  635:01        1227
  636:01        1142
  637:01        1216
  638:01        1259
  639:01        1299
```

640:01	1232
641:01	1117
642:01	1155
643:01	1232
644:01	1083
645:01	1020
646:01	1394
647:01	1196
648:01	1148
649:01	1083
650:01	1189
651:01	1133
652:01	1034
653:01	1157
654:01	1034
655:01	1097
656:01	1299
657:01	1157
658:01	1130
659:01	1155
660:01	1349
661:01	1232
662:01	1103
663:01	1103
664:01	1083
665:01	1027
666:01	1166
667:01	1148
668:01	1250

669:01	1155
670:01	1047
671:01	1054
672:01	1018
673:01	1189
674:01	1126
675:01	1250
676:01	1297
677:01	1178
678:01	1043
679:01	1103
680:01	1250
681:01	1272
682:01	1169
683:01	1004
684:01	1083
685:01	1164
686:01	1124
687:01	1027
688:01	995
689:01	1169
690:01	1270
691:01	1011
692:01	1247
693:01	1101
694:01	1004
695:01	1004
696:01	1065
697:01	1223

698:01	1184
699:01	1216
700:01	1180
701:01	1142
702:01	1277
703:01	1206
704:01	1076
705:01	1076
706:01	1189
707:01	1121
708:01	1178
709:01	1031
710:01	1076
711:01	1178
712:01	1209
713:01	1022
714:01	1220
715:01	1070
716:01	1126
717:01	1058
718:01	1216
719:01	1358
720:01	1184
721:01	1083
722:01	1097
723:01	1119
724:01	1097
725:01	1097
726:01	1153

727:01	1153
728:01	1151
729:01	1151
730:01	1151
731:01	1184
732:01	1097
733:01	1043
734:01	1043
735:01	1002
736:01	1152
737:01	1097
738:01	1034
739:01	1002
740:01	989
741:01	1092
742:01	1115
743:01	1115
744:01	1047
745:01	1040
746:01	1038
747:01	1085
748:01	1126
749:01	1058
750:01	1067
751:01	1115
752:01	1263
753:01	1124
754:01	1110
755:01	1097

756:01	1097
757:01	1157
758:01	1000
759:01	991
760:01	995
761:01	1013
762:01	1007
763:01	971
764:01	971
765:01	980
766:01	993
767:01	1043
768:01	1097
769:01	982
770:01	971
771:01	971
772:01	1065
773:01	1022
774:01	1029
775:01	989
776:01	1029
777:01	995
778:01	982
779:01	1090
780:01	980
781:01	971
782:01	957
783:01	989
784:01	966

785:01	989
786:01	1022
787:01	1074
788:01	1110
789:01	1110
790:01	1061
791:01	1151
792:01	1128
793:01	1074
794:01	1043
795:01	1034
796:01	1074
797:01	966
798:01	1027
799:01	1029
800:01	1034
801:01	1065
802:01	989
803:01	1034
804:01	1002
805:01	1128
806:01	1178
807:01	1097
808:01	1142
809:01	1466
810:01	1097
811:01	1137
812:01	1097
813:01	1259

814:01	1313
815:01	1173
816:01	1169
817:01	1173
818:01	1088
819:01	1191
820:01	1146
821:01	1160
822:01	1142
823:01	1128
824:01	1169
825:01	1162
826:01	1115
827:01	1164
828:01	1088
829:01	1079
830:01	1083
831:01	1043
832:01	1110
833:01	1092
834:01	1110
835:01	1047
836:01	1076
837:01	1110
838:01	1043
839:01	1103
840:01	1034
841:01	1074
842:01	1052

843:01	1011
844:01	1097
845:01	1092
846:01	1110
847:01	1115
848:01	1097
849:01	1196
850:01	1115
851:01	1162
852:01	1151
853:01	1142
854:01	1126
855:01	1108
856:01	1187
857:01	1191
858:01	1153
859:01	1254
860:01	1187
861:01	1196
862:01	1331
863:01	1412
864:01	1349
865:01	1290
866:01	1211
867:01	1232
868:01	1166
869:01	1124
870:01	1146
871:01	1079

```
diff(center) nile / cnile

@bjautofit(pmax=10,qmax=10) cnile
```

AIC analysis of models for series CNILE

MA

AR	0	1	2	3	4	5	6	7	8	9	10
0	2970.6959	2921.2743	2914.8334	2912.7899	2908.0178	2899.2567	2901.0289	2901.1309	2894.7245	2896.2238	2894.5270
1	2972.6959	2892.2803	2888.7582	2890.0017	2891.4626	2893.4617	2893.4044	2894.3237	2896.3222	2895.0891	2894.4733
2	2903.1668	2889.9935	2890.3157	2891.7676	2893.4624	2895.1196	2894.9661	2896.3233	2896.3555	2894.8735	2896.1532
3	2895.8469	2889.3935	2891.2107	2891.8621	2889.8453	2887.7432	2898.6062	2901.1044	2890.8151	2892.6474	2893.9366
4	2890.7725	2890.9802	2892.9721	2890.4777	2895.6908	2894.0509	2890.2037	2890.8276	2902.0466	2894.4542	2896.4333
5	2890.8607	2892.4134	2888.1112	2887.0989	2894.1950	2889.3421	2886.4625*	2916.6542	2892.4229	2897.0004	2897.9030
6	2892.6571	2894.3819	2887.2058	2892.1534	2908.4757	2915.6049	2904.6687	2899.9135	2899.1808	2906.3855	2902.0129
7	2892.9749	2894.6620	2896.4342	2895.3395	2896.7452	2889.1461	2902.9879	2887.3730	2893.9933	2898.4374	2927.8723
8	2894.2911	2894.2839	2890.2677	2911.8523	2893.6490	2895.6262	2893.7176	2893.2514	2892.2732	2895.1678	2898.6734
9	2895.1436	2895.5864	2898.2373	2893.8419	2905.1811	2905.9628	2915.7516	2896.5580	2914.7075	2923.7791	2928.5822
10	2895.2207	2896.6978	2892.1059	2893.7539	2897.6910	2893.0241	2894.6637	2922.9200	2896.0643	2896.8553	2898.3841

```
boxjenk(maxl,ar=5,ma=3,demean) nile
```

Box-Jenkins - Estimation by ML Gauss-Newton

Convergence in 15 Iterations. Final criterion was 0.0000023 <= 0.0000100

Dependent Variable NILE

Annual Data From 622:01 To 871:01

Usable Observations	250
Degrees of Freedom	242
Centered R^2	0.3315989
R-Bar^2	0.3122650
Uncentered R^2	0.9955066
Mean of Dependent Variable	1119.0360000
Std Error of Dependent Variable	92.2458448

```
Standard Error of Estimate          76.4992849
Sum of Squared Residuals          1416218.0219
Log Likelihood                      -1435.5494
Durbin-Watson Statistic                 1.9841
Q(36-8)                                18.9895
Significance Level of Q              0.8984611

    Variable                         Coeff      Std Error      T-Stat      Signif
*********************************************************************************
1.  AR{1}                         -0.324434186  0.170260121     -1.90552  0.05789755
2.  AR{2}                         -0.061167826  0.147197227     -0.41555  0.67810745
3.  AR{3}                          0.633074471  0.092652506      6.83278  0.00000000
4.  AR{4}                          0.069262385  0.116922925      0.59238  0.55415162
5.  AR{5}                          0.248128667  0.075352608      3.29290  0.00113979
6.  MA{1}                          0.703020655  0.173727366      4.04669  0.00006992
7.  MA{2}                          0.351410612  0.201954147      1.74005  0.08312123
8.  MA{3}                         -0.417882494  0.170499232     -2.45093  0.01495710

declare frml[complex] transfer
dec vect a(2) b(2)
nonlin a b d ivar
* Construct the transfer function of the moving average polynomial. This
* example includes one MA lag and one AR lag in addition to the
* fractional difference. The %conjg function on the final term is
* necessary in the next function since the * operator conjugates the
* right operand, and we want a straight multiply.
frml transfer = (1+b(1)*%zlag(t,1)+b(2)*%zlag(t,2))/$
   ((1-a(1)*%zlag(t,1)-a(2)*%zlag(t,2))*%conjg((1-%zlag(t,1))**D))
```

```
Run an AR(1) to get a guess for the innovation variance.
*
linreg(noprint) nile
# constant nile{1}

linreg nile
# constant nile{1}

Linear Regression - Estimation by Least Squares
Dependent Variable NILE
Annual Data From 623:01 To 871:01
Usable Observations                    249
Degrees of Freedom                     247
Centered R^2                     0.2391652
R-Bar^2                          0.2360849
Uncentered R^2                   0.9948669
Mean of Dependent Variable    1118.8835341
Std Error of Dependent Variable  92.4000686
Standard Error of Estimate       80.7597286
Sum of Squared Residuals      1610967.0408
Regression F(1,247)                77.6434
Significance Level of F          0.0000000
Log Likelihood                  -1445.7898
Durbin-Watson Statistic             2.1100

    Variable                        Coeff      Std Error      T-Stat      Signif
************************************************************************************
1.  Constant                    571.52478980  62.32881338      9.16951  0.00000000
2.  NILE{1}                       0.48906390   0.05550260      8.81155  0.00000000
```

```
compute a=||.3,.3||,b=||0.,.5||,d=.5,ivar=%seesq

* Use double the recommended number of ordinates. Send the data to the
* frequency domain and compute the periodogram.

nonlin a b d=.4 ivar
compute nords = %freqsize(871:1)*2
compute nords = 871:1
compute nobs  = 871:1
freq 3 nords
rtoc
# cnile
# 1
fft 1
cmult(scale=1.0/nobs) 1 1 / 2
frml periodogram = %real(%z(t,2))
frml likely    = trlambda=transfer, (float(nobs)/nords)*$
  (-.5*log(2*%pi)-.5*log(ivar)-log(%cabs(trlambda))-$
     .5/ivar*periodogram/%cabs(trlambda)^2)
maximize(iters=400,method=bfgs,pmethod=simplex,piters=5) likely 1 nords


MAXIMIZE - Estimation by BFGS
Convergence in    37 Iterations. Final criterion was  0.0000066 <=  0.0000100
Annual Data From 622:01 To 871:01
Usable Observations                        249
Skipped/Missing (from 250)                   1
```

```
Function Value                          -1431.2490

    Variable                              Coeff       Std Error      T-Stat      Signif
*************************************************************************************
1.  A(1)                               0.174032     0.059623      2.91886  0.00351314
2.  A(2)                              -0.969143     0.026204    -36.98412  0.00000000
3.  B(1)                              -0.188672     0.088101     -2.14154  0.03223090
4.  B(2)                               0.926565     0.044231     20.94833  0.00000000
5.  IVAR                            5856.201070   466.749763     12.54677  0.00000000

*
* Dividing the FT of the series by the MAR transfer function gives the
* FT of the residuals. Inverse transform and send the relevant part back
* to the time domain.

cset 3 = %z(t,1)/transfer(t)
ift 3
ctor 622:1 871:1
# 3
# resids
* Check out the correlation of residuals. We skip the first few
* residuals in this as they are likely to be very large with a
* correspondingly large variance.
corr(qstats,span=24) resids


Correlations of Series RESIDS
Annual Data From 622:01 To 871:01
```

Autocorrelations

1	2	3	4	5	6	7	8	9	10
-0.02355	-0.05504	-0.00361	0.00483	0.06090	-0.00675	0.04688	0.01859	-0.12544	0.11648
11	12	13	14	15	16	17	18	19	20
-0.00198	-0.07840	0.03889	-0.06503	0.02644	0.04556	-0.02209	0.00368	0.03637	-0.01075
21	22	23	24	25	26	27	28	29	30
-0.02501	0.10804	-0.00143	-0.03237	-0.05898	0.05709	-0.10533	-0.09476	0.10523	0.02747
31									
-0.05217									

Ljung-Box Q-Statistics

Lags	Statistic	Signif Lvl
24	18.342	0.786101

References

1. P.J. Brockwell and R.A. Davis,(2002), *Introduction to time series and forecasting*, 2 ed, Columbia university

2. RATS Procedure and Example Files: https://estima.com/ARCH-GARCH.shtml